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Effects of Errors on Decoupled Control Systems

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SUMMARY

A study to determine the effects of various error sources on a decoupled control system was performed in connection with the longitudinal control system on a simulated externally blown jet-flap STOL aircraft. The system employed the throttle, horizontal tail, and flaps to decouple the forward velocity, pitch angle, and flight-path angle. The errors considered were (1) imperfect knowledge of airplane aerodynamic and control characteristics, (2) imperfect measurements of airplane state variables, (3) change in flight conditions, and (4) lag in the airplane controls and in the engine response. The error due to change in flight conditions results from the use of constant gains to represent a range of flight conditions.

Despite the large errors employed in the study, the effects of the various errors on the decoupling process were generally minor. Because changes in flight conditions had little effect, constant gains in a decoupled control system for certain flight regimes would be feasible, thereby avoiding the need for an onboard computer.

The largest effects on decoupling were caused by control lag and by change in flight condition during speed-command maneuvers wherein significant coupling in the flight-path angle was produced. However, these effects could be eliminated by retrimming the flight-path angle when large speed changes were required, such as in high decelerating approaches. In addition, lag effects could be eliminated by including the lag in the design of the decoupled control system.

Except for the effects on the flight-path angle during speed-command maneuvers, the largest effect of any error source generally occurred in the commanded response quantity; the other responses were materially unaffected, as were the control requirements. It was shown that if a pilot could recognize these discrepancies in the command response, he could compensate with an additional command input, with only small increased effects on the decoupling process.

INTRODUCTION

Decoupling is one of the more sophisticated forms of automatic control devised in recent years. (See, for example, refs. 1 to 4.) The method of airplane decoupled control can be briefly stated in three steps:

(1) Choose the airplane response quantities to be controlled.

(2) Provide the pilot with one input channel for each quantity.

(3) Design the control system so that an input command in one channel gives desirable response to the command input and no response in the other channels.

An automatic control system of this type would have many practical applications as, for example, use in strafing fast-moving targets where constant pitch angle must be maintained during changes in flight-path angle and speed. Decoupled control would also be highly desirable for STOL aircraft operation where severe control requirements may exist during approach and landing because of excessive coupling between the longitudinal flight variables, flight-path angle, forward speed, and pitch attitude. The control problem is further complicated for this powered lift airplane because a sizable flight-path-angle response is associated with the throttle; furthermore, if jet engines are the power plants, a considerable time lag occurs between the throttle movement and the change in engine thrust.

Application of decoupled control systems to certain types of aircraft has been studied extensively over the past few years. (See, for example, refs. 5 to 8.) Knowledge of the effects of various error sources is necessary for future practical applications. Therefore, this paper investigates the magnitude of these effects on the decoupling process. The effects studied include the effect of errors in the (1) knowledge of the airplane aerodynamic and control characteristics, (2) measurements which operate on the feedback gains, (3) use of constant gains when the flight conditions are actually changing, and (4) lag in the control actuators.

The present study was conducted using a simulated STOL aircraft for decoupled longitudinal control in a landing approach condition. The computer program reported in reference 9 was used for the decoupling calculations. This program provides complete decoupling because both the transient and steady-state airplane responses are decoupled. In addition to the decoupling, the program permits the choice of the zeros and poles of each input-output transfer function by specifying the polynomial coefficients of the numerator and denominator.

SYMBOLS

A matrix defined by aircraft stability coefficients

$\hat{A}$ matrix $= A + BF$

a element of A matrix

B matrix defined by aircraft-control coefficients

b element of B matrix

C matrix relating desired output vector to state vector

C_m pitching-moment coefficient

C_W aircraft weight in coefficient form $\left(-\frac{2mg}{\rho V^2 S}\right)$

C_X longitudinal-force coefficient

C_Z	normal-force coefficient
$\bar{c}$	mean aerodynamic chord, m
F	matrix of feedback gains used in decoupled controller (see appendix A)
f	element of F matrix
G	matrix of feedforward gains used in decoupled controller (see appendix A)
g	acceleration due to gravity, m/sec^2
I	identity matrix
I_Y	moment of inertia about Y-axis, $kg-m^2$
i,j,k, l,m,n,r	indices
m	mass of airplane, kg
q	pitch rate, rad/sec
S	wing area, m^2
s	Laplace operator
t	time, sec
u	velocity component along X-axis, m/sec
$\bar{u}$	m-vector of control variables
V	equilibrium or trim airspeed, m/sec
$\bar{v}$	m-vector of commanded pilot inputs
X,Y,Z	body-axis system
$\bar{x}$	n-vector of state variables
$\bar{y}$	m-vector of state variables to be controlled in decoupled manner
α	angle of attack, deg
γ	flight-path angle, deg
Δ	incremental change from equilibrium or trim condition
δ_f	deflection of trailing-edge flaps, deg

δ_t horizontal-tail deflection, deg

δ_{th} throttle deflection

ζ short-period damping ratio for closed-loop (decoupled) airplane

ζ_t damping ratio for horizontal-tail servo

ζ_1 damping ratio in speed response

ζ_3 damping ratio in γ response

θ pitch angle, deg

ρ air density, kg/m^3

τ_f flap response time constant, sec

τ_t horizontal-tail response time constant, sec

τ_{th} thrust response time constant, sec

τ_1 speed response time constant, sec

τ_3 flight-path-angle response time constant, sec

ω_n longitudinal short-period natural (undamped) frequency for closed-loop (decoupled) airplane, rad/sec

ω_t natural frequency of horizontal-tail servo, rad/sec

ω_1 natural frequency in speed response, rad/sec

ω_3 natural frequency in γ response, rad/sec

Aircraft stability and control coefficients:

$$C_{X_{\delta_f}} = \frac{\partial C_X}{\partial \delta_f} \qquad C_{Z_{\delta_f}} = \frac{\partial C_Z}{\partial \delta_f} \qquad C_{m_{\delta_f}} = \frac{\partial C_m}{\partial \delta_f}$$

$$C_{X_{\delta_{th}}} = \frac{\partial C_X}{\partial \delta_{th}} \qquad C_{Z_{\delta_{th}}} = \frac{\partial C_Z}{\partial \delta_{th}} \qquad C_{m_{\delta_{th}}} = \frac{\partial C_m}{\partial \delta_{th}}$$

$$C_{X_{\delta_t}} = \frac{\partial C_X}{\partial \delta_t} \qquad C_{Z_{\delta_t}} = \frac{\partial C_Z}{\partial \delta_t} \qquad C_{m_{\delta_t}} = \frac{\partial C_m}{\partial \delta_t}$$

$$C_{X_u} = \frac{\partial C_X}{\partial \frac{u}{V}} \qquad C_{Z_u} = \frac{\partial C_Z}{\partial \frac{u}{V}} \qquad C_{m_u} = \frac{\partial C_m}{\partial \frac{u}{V}}$$

$$C_{X_\alpha} = \frac{\partial C_X}{\partial \alpha} \qquad C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha} \qquad C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$$

$$C_{X_q} = \frac{\partial C_X}{\partial \frac{q\bar{c}}{2V}} \qquad C_{m_q} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}}$$

$$C_{X_{\dot{\alpha}}} = \frac{\partial C_X}{\partial \frac{\dot{\alpha}\bar{c}}{2V}} \qquad C_{m_{\dot{\alpha}}} = \frac{\partial C_m}{\partial \frac{\dot{\alpha}\bar{c}}{2V}}$$

Superscripts:

-1 matrix inverse

' perturbations from equilibrium

Subscripts:

0 equilibrium or trim condition

c commanded by pilot

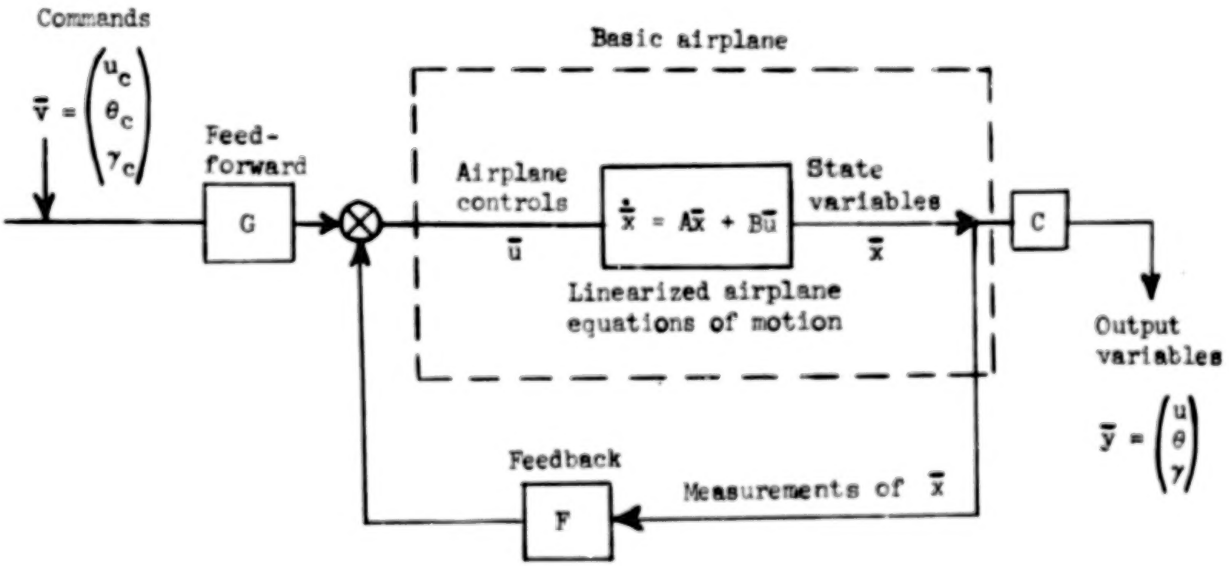
Dot over symbol denotes differentiation with respect to time.

DESCRIPTION OF SIMULATED AIRPLANE

The STOL aircraft simulated in this study is the clustered-engine airplane aerodynamically described in references 10, 11, and 12. The airplane (fig. 1) is a high-wing jet transport with four high bypass ratio turbofan engines. The four engines yield a maximum total thrust of 147 057 N. The pertinent physical characteristics of the simulated aircraft including maximum control-surface deflection and deflection rate are presented in table I. The aerodynamic characteristics are presented in table II. Values are shown for two flight conditions, $\alpha = 10^{\circ}$ and $\alpha = 5^{\circ}$. In both cases, the nominal speed is 30.48 m/sec and the thrust coefficient is 1.87 (one-half power). Where $\alpha = 10^{\circ}$, two values are changed which account for thrust effects on C_Z and C_m that were not included in the standard set of coefficients.

DECOUPLED CONTROL

The approach taken in providing independent longitudinal control of pitch angle, flight-path angle, and forward velocity is depicted in sketch (a).



Sketch (a)

The feedforward gain matrix G operates on the pilot command inputs in order to provide output solely in the commanded airplane responses. Both gain matrices F and G operate to decouple and yield desired responses. The state variables must be continuously measured and operated on by the feedback gain matrix F to move the airplane controls automatically in order to maintain the desired decoupled responses. The C matrix allows any combination of the state variables to be selected as the output quantities to be decoupled. Figure 2 illustrates the effectiveness of decoupled control. The results in figure 2(a) pertain to a horizontal-tail step input of about 2° for the unaugmented (no decoupled control) system. Not only is the coupling between the three response quantities apparent, but the intended response in θ is sluggish. For the decoupled system, figure 2(b) shows how the controls automatically move to maintain the u and γ responses at 0 for a pitch command of about 3°. Also, the θ response has been improved by selecting $\omega_n = 2$ rad/sec and $\zeta = 0.7$ as the closed-loop (decoupled) dynamic characteristics.

The most sophisticated implementation of decoupled control would employ an onboard computer to determine the time-varying adaptive gain matrices F and G. However, this degree of complexity may not be required if the decoupled control is limited to certain flight regimes, such as the approach and landing phase. In the present study these F and G gain matrices are treated as constants, thus eliminating the need for an onboard computer. There are various methods of determining the required F and G gains for longitudinal decoupled control. One method, which both simulation and flight testing (ref. 8) have proven to be practical, is devised in reference 7 for decoupled control of the steady-state condicions. Other methods have been developed for complete decoupling in order

to include the transient conditions. The method of determining F and G in reference 3, along with the computer program in reference 9, is used for the present study.

A general development of the method for complete decoupling is given in appendix A. Because of its clarity and simplicity, Rekasius' method (ref. 1) for decoupling analysis was chosen instead of the abstract geometric and algebraic methods of most decoupling analyses.

METHOD OF ANALYSIS

All cases in the present analysis pertain to decoupled control of the longitudinal response quantities u, θ, and γ. The values of the coefficients for the A and B matrices, as determined from equations (A1) to (A3) and from table II, are presented in table III. Unless otherwise noted, the results in this paper pertain to $\alpha = 10^{\circ}$ (see table III(a)) which has been denoted the standard case for this analysis.

The analysis presented in this paper is quantitative in nature and is intended to give an overall indication of the effects of various error sources on the decoupling process. The errors considered were (1) imperfect knowledge of airplane dynamics, (2) imperfect measurements of airplane state variables, (3) change in flight conditions, and (4) lag in the airplane controls and in the engine response.

The results of the study were obtained from computer time-history simulations of step-command inputs using values of the F and G gain matrices obtained from the computer program of reference 9. Table IV presents typical values of the gain matrices and the airplane design response characteristics used in the analysis.

DISCUSSION OF RESULTS

The results of the investigation are presented primarily as computer-simulated time histories of airplane responses and control motions. Time histories pertaining to various types of errors are compared with the nominal (no error) time histories to show the magnitude of the effects of the errors imposed on the system.

Effect of Imperfect Knowledge of Airplane Characteristics

In order to compute the constant gains for the decoupled control system designers need the airplane aerodynamic and control characteristics, as reflected by the A and B matrices (appendix A). These characteristics are determined from the airplane stability and control derivatives which must be obtained from wind-tunnel tests, flight tests, or theoretical calculations. There is always some question about the accuracy of these values and, hence, about the computed gains required for decoupling. Nonlinear effects also account for error in the A and B matrices. This error is introduced by

linearizing the airplane equations of motion in the decoupling calculations (appendix A).

Effects on airplane decoupled responses.- Tables V and VI are presented as a quantitative summary of the effects of errors in the different A and B elements on decoupling. In each table the stability or control derivatives mainly responsible for each element are given in the different blocks of the A or B matrix found at the top of the tables. As noted by the zero $b_{2,1}$ and $b_{3,1}$ elements in table VI, the results apply to the standard case.

For the A matrix, the elements were reduced to zero, one at a time, to determine their individual effects on decoupling. Decoupling was more sensitive to errors in the B-matrix elements; hence, the elements were reduced to one-half their nominal values only. In all cases the values of the F and G matrices remained the same. The results of changing the values of the elements are shown in the charts at the bottom of the tables. For example, for an error in $a_{2,2}$, the chart shows that decoupling is not lost for a pitch command, but that an overshoot of 20 percent in the commanded pitch value occurs. (The terms "overshoot" and "undershoot" refer to the ensuing steady-state error during a command maneuver.) The results for pitch command are similar for an error in $a_{2,3}$. This overshoot effect is not serious, as the pilot can compensate for this effect, as shown in subsequent examples. For an error in $a_{3,3}$, the chart indicates that decoupling is lost in the flight-path-angle response for a pitch command; decoupling is not significantly affected for the other two commands. For an error in the elements in the fourth column of the A matrix, the charts show no significant effect on decoupling for either a θ or γ command, and only small effects for a speed command. For the blocks showing partial decoupling, the effects are not considered serious because of the severe errors that were imposed on the A and B elements.

Further tests whose results are not presented in the tables were made with $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$. Changing $b_{3,1}\left(C_{Z_{\delta_{th}}}\right)$ to one-half its nominal value had no significant effect on decoupling. A similar change in $b_{2,1}$, however, destroyed decoupling in γ for θ and u commands and caused a large (70-percent) overshoot in γ for a γ command. Effects on the control motions were not excessive in any case.

Effects on control motions.- Some effects of the A- and B-matrix errors on the control motions, as well as on the commanded responses, are shown in figures 3 to 5. The effects on the controls may be important because of control displacement or rate limits. In regard to A-matrix errors (fig. 3), the flap and throttle are the controls most affected. The maximum flap deviation from the nominal is about 15 percent of full deflection, and the maximum deviation from the nominal for the throttle is about 20 percent. (Fig. 3(b) shows a small overshoot in γ for an error in $a_{2,3}$, not noted in table V because decoupling in θ has been completely lost.) B-matrix errors which affect decoupling show little effect on the controls for a θ command (fig. 4(a)). For a γ command, an error in $b_{4,1}$ is apparently the most serious with regard to the controls; the steady-state throttle deflection increases over 150 percent of nominal and would exceed its limit value. Figure 5(a) is presented to show the amount of coupling in θ for an error in $b_{2,2}$. The

commanded value of γ, however, is still achieved ($\gamma = \theta - \alpha$). For complete decoupling, $\gamma = -\alpha$.

As shown in figure 5(a), an error of 1° is obtained in θ. If the pilot could recognize this error, he could compensate for it by employing a θ command of 1.12° as shown in figure 5(b). The additional 12-percent command is needed to account for the undershoot due to the $b_{2,2}$ error. (See table VI.) Also, for a pitch command, table VI shows that the $b_{2,2}$ error produces no significant effect on γ. By using the θ command in figure 5(b), the θ error is reduced to zero, with the command value of 6° still being maintained for γ.

Summary of effects.- The overall effects of errors in the A and B matrices can be summarized as follows: The more important A-matrix elements, as far as sensitivity to decoupling is concerned, appear to be $a_{4,1}$ (C_W) and those in the third column (α derivatives). For the B matrix, errors in $b_{4,1}\left(C_{X_{\delta_{th}}}\right)$ and $b_{4,3}\left(C_{X_{\delta_f}}\right)$ have a coupling effect on u for θ and γ commands, while the $b_{2,2}\left(C_{m_{\delta_t}}\right)$ element essentially affects only θ during a γ command. Also, for the case where the nominal value of $b_{2,1}$ was not zero, errors in this element produced coupling in γ for θ and u commands.

For A-matrix errors, effects on decoupling as well as changes in the control motions were essentially linearly related to the magnitude of the errors. This linearity was not the case for B-matrix errors because of their highly nonlinear effect on the decoupling process. In some instances, changing the B elements to zero magnitude prevented decoupling, whereas the one-half nominal magnitude error had only a marginal effect. These results indicate that a better knowledge of the B matrix than of the A matrix is required in designing adequate decoupled control systems.

Effect of Imperfect Measurements of State Variables

In the decoupling process the feedback gain matrix F operates on the measured values of the airplane state variables θ, $\dot{\theta}$, α, and u. In any practical design, errors in these measurements can have an effect on decoupling. In extreme cases these errors can become quite large owing to instrument malfunction. The measurement-error effects were investigated, both individually and in combination. For the present analysis, these measurement errors were simulated by changing the values in certain columns of the F and BF matrices by various amounts. Equations (A12) and (A14) are repeated here to show that these changes are equivalent to prescribing errors in the various measured response quantities, $\bar{x}$.

$$\bar{u} = F\bar{x} + G\bar{v}$$

$$\dot{\bar{x}} = (A + BF)\bar{x} + BG\bar{v}$$

For example, according to the matrix which indicates the arrangement of the equations of motion as given in table III, a 20-percent increase in the values

of the third columns of F and BF would represent a 20-percent error in the α measurement.

Individual effects on decoupling.- Examples of the effects of errors in the various measurements are presented as time-history comparisons in figures 6 to 9. These effects are summarized in table VII. The table clearly indicates that measurement inaccuracy in the pitch rate $\dot{\theta}$ is not important. Also, the table shows that the dominant effect of the errors in each of the other three variables occurs in the particular commanded response quantity in which the error exists. For example, with measurement error in α (which corresponds to $-\gamma$ in the decoupled system), a large effect on γ and small effects on θ and u are noted for a γ-command maneuver and little or no effect on θ, γ, and u for either a θ or u command. Time-history examples of these effects are presented in figure 6. For the cases in figure 6 the effects in the airplane responses and control motions appear to be essentially linear with respect to the magnitude of α-measurement error.

With regard to velocity-feedback error, table VII shows that this error is important only in a u-command maneuver. An example time history of this type of maneuver is shown in figure 7. The figure indicates that the control motions are not significantly affected.

The time histories in figure 8 illustrate the effect of θ-measurement error for a θ-command maneuver. The major effect of the error is in the θ response. Two nominal cases are included in the figure to show how the controls react for two different sets of decoupled responses. In the one case u, θ, and γ are decoupled (controlling γ), and in the other, u, θ, and α are decoupled (controlling α). In order to avoid confusion, nominal values of γ and α are not plotted in figure 8. For the controlled γ case, nominal α, of course, equals θ and in the controlled α case, nominal γ also equals θ. It is shown that the measurement error does not affect the α response; however, there is a small effect in γ inasmuch as γ depends on θ in the equations of motion.

Combined effects on decoupling.- The effects of combined measurement errors are illustrated in table VIII. The results in the table show primarily that combined errors have about the same effect as individual errors. For example, in table VIII(b) the effect of combined errors is the same as the effect of an α error only; that is, the effect of adding other errors is insignificant. In table VIII(a), for a θ command, only the θ-measurement error is important. The effects of combined measurement errors are not shown for u-command maneuvers because they are negligible. Thus, for a u command, only the u-measurement error is important.

Linearity characteristics of error effects.- During the investigation it was noticed that the effects of θ-measurement errors were nonlinear. An example of this nonlinearity is apparent in table VIII(a) for positive and negative θ error. Another example is shown in time-history form in figure 9. It also became apparent that the error in the u or γ commanded response quantity depends on the magnitude of the nominal value of the corresponding element in the A matrix. For example, for speed response the pertinent element would be $a_{4,4}$ (according to the arrangement shown in table III), since this element

corresponds to the term in the velocity equation ($\dot{u}$ row) that would be affected by u-measurement error. For Y response the pertinent element would be $a_{3,3}$. The linearity characteristics of the error effects were analyzed by developing the following equations.

From the decoupled equations ((A13) and (A14))

$$\bar{y} = C\bar{x}$$

$$\dot{\bar{x}} = A\bar{x} + BF\bar{x} + BG\bar{v}$$

let $y_i = x_k$ where x_k is the commanded response. Then the kth augmented equation of motion can be written as

$$\dot{x}_k = \sum_{j=1}^{n} a_{kj}x_j + \sum_{r=1}^{m}\sum_{j=1}^{n} b_{kr}f_{rj}x_j + \sum_{\substack{r=1\\ l=1}}^{m} b_{kr}g_{rl}v_l \qquad (1)$$

The steady-state error in the response x_k caused by an error in the measurement of x_k can be determined as in the following discussion. For steady state, $\dot{x}_k = 0$. Assuming a relative error E in the measured x_k gives the following expression for the measured quantity:

$$x_{k,meas} = (1 + E)x_k$$

Substituting this expression into the term in equation (1) containing the feedback matrix F yields

$$\left[a_{kk} + (1 + E)\left(\sum_{r=1}^{m} b_{kr}f_{rk}\right)\right]x_k = -\sum_{\substack{r=1\\ l=1}}^{m} b_{kr}g_{rl}v_l - \hat{a}_{kj}x_j \qquad (j \neq k) \qquad (2)$$

where the term $\hat{a}_{kj}x_j$ represents the summation of all response-quantity terms other than the commanded response. Let R represent the right-hand side of equation (2). For no error in measuring x_k, the equation for the nominal value of x_k can be written as

$$\left(a_{kk} + \sum_{r=1}^{m} b_{kr}f_{rk}\right)x_{k_n} = R \qquad (3)$$

where x_{k_n} represents the nominal value.

Replacing a_{kk} with a and $\sum_{r=1}^{m} b_{kr}f_{rk}$ with bf gives the relative error in the steady-state response caused by measurement error in x_k only

(using eq. (3) for nominal and eq. (2) for actual):

$$\frac{x_k - x_{k_n}}{x_{k_n}} = \frac{\dfrac{R}{a + (1 + E)bf} - \dfrac{R}{a + bf}}{\dfrac{R}{a + bf}}$$

$$\frac{x_k - x_{k_n}}{x_{k_n}} = \frac{a + bf}{a + bf + Ebf} - 1$$

$$\frac{x_k - x_{k_n}}{x_{k_n}} = \frac{-Ebf}{a + bf + Ebf}$$

$$\frac{x_k - x_{k_n}}{x_{k_n}} = \frac{-E}{1 + E + \dfrac{a}{bf}}$$

$$\frac{x_k - x_{k_n}}{x_{k_n}} = \frac{-E}{E + c} \tag{4}$$

where

$$c = 1 + \frac{a}{bf}$$

An example of results determined from equation (4) is presented in figure 10. It is of interest to note that the curve for $c = 1$, which represents the maximum possible error, applies to θ under any condition inasmuch as the pertinent a element is always zero. Examination of figure 10 reveals the nonlinear characteristics of the response error with regard to both measurement error and the nominal value of the pertinent element of the unaugmented A matrix. A noticeable difference in these characteristics exists between positive and negative measurement error. Negative measurement errors have the largest effect, especially in the case of the θ response, where $c = 1$. Positive measurement error refers to a bias error in the measurement which produces a decrease in the absolute magnitude of the response quantity, and conversely for negative measurement error. (For example, see fig. 9.)

Because the magnitude of the commanded response error depends on the nominal value of a, the error in the γ or u commanded response could be reduced by altering the A matrix with a change in the aircraft aerodynamic design.

Compensating for measurement error.- The results of the overall error analysis have shown that measurement error in a given response quantity affects the

commanded response of that particular quantity primarily, with only small amounts of coupling in the other response quantities. In general, this effect on the commanded response would not be critical in a decoupled control system because the pilot would normally recognize the effect on the response he is commanding and could compensate for this error. One such case is shown in figure 11. The short-dashed line in the figure duplicates the one in figure 8 and shows a 28-percent error in the pitch response. In anticipating the error and by compensating with an additional degree of pitch command, as represented by the long-dashed curve, the pilot can obtain the originally desired pitch angle of 3°. The additional effect on the controls and on the coupling of the other airplane response quantities is minor.

Effect of Change in Flight Conditions

In the design of a decoupled control system the F and G gain matrices are determined by the values of the A and B matrices which, in turn, depend on the airplane stability and control derivatives. (See appendix A.) These derivatives vary with the airplane flight conditions: speed, angle of attack, and thrust coefficient, primarily. As previously noted, a decoupled control system employing a set of constant gains corresponding to a certain set of flight conditions would be advantageous in any practical implementation. This section investigates the feasibility of using constant gains to approximate a decoupled control system operating under varying flight conditions. The results give an indication of the magnitude of the flight condition effects on the decoupled airplane responses as well as on the required control motions.

Effect of speed.- The effect of changes in speed on the A and B matrices is shown in table IX. These changes in the elements of A and B affect the decoupling process inasmuch as the gains are calculated only for nominal speed u_0. In order to determine the effects of speed, the A and B elements were changed according to the ratios in table IX (α was assumed not to change; hence, the derivatives did not change) whereas the F and G gain matrices that were calculated for the original nominal speed of 30 m/sec were left unchanged. Time histories were then simulated for individual step commands in the airplane responses θ, γ, and u. An example is shown in figure 12 and the results of the analysis are summarized in table X.

Effects on airplane decoupled responses.- As shown by the data in table X, speed has an insignificant effect on the decoupling process for θ and γ commands. The commanded values are generally obtained with little coupling in the other response quantities. Some noticeable effect, however, is apparent for the speed-command maneuver. The major effects are seen to be the 1.4° coupling in γ and, in the case where $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$, the large error in the commanded u response. (It is of interest to note that a 1.5-m/sec speed change for the unaugmented airplane produced a γ change of 4.5°.) This case had two additional nonzero elements in the B matrix (table III) which resulted in different F and G gain matrices (table IV). The speed effects on the u-command maneuver are quite different for this case, the difference being caused by the change in the nominal values for the F gain matrix. The fact that speed

effect depends on the nominal F matrix signifies that this effect would vary for different types of aircraft.

It is important to state that the pilot could compensate for the large error in the u response during a speed-command maneuver in the same manner described in the previous section where measurement error resulted in large discrepancies in the commanded responses.

The pilot could compensate for the large coupling in γ in a manner similar to that shown in figure 5(b). Table X shows that a γ command for correcting γ would have no coupling effect on u or θ. In addition, simulator studies in reference 7 show that excessive coupling in γ during large speed changes (high decelerating approaches) was eliminated by retrimming the flight-path angle, either automatically or by the pilot.

Effects on control motions.- The results of flight-condition change on the control motions are not included in table X. There were only slight changes noted in the control motions for all cases except the speed-command maneuver. A sample case is shown in figure 12 where the speed has been changed from 30.48 m/sec to 60.96 m/sec. The speed has the largest effect on the flap, which is a change of about 30 percent from the nominal value. Of course, in all cases the controls as well as the response quantities exhibit a noticeable change from nominal in the time response as illustrated in figure 12. This change is caused by the fact that the $\hat{A}$ matrix has been altered by changing the A-matrix stability derivatives, which is equivalent to changing the augmented airplane dynamic characteristics.

Effect of angle of attack.- The effects of changing the angle of attack α from 10° to 5° are shown in figure 13 for step commands in θ, γ, and u. Changes in α affect the airplane stability and control derivatives which, in turn, affect the A and B matrices. (See tables II and III.) As shown in table III, a change of 5° in α produces changes of about 5 to 10 percent in the A and B coefficients. The effects of the α change on decoupling were investigated in the same manner as the speed effects.

The time-history examples in figure 13 show that an α change had only minor effects on the decoupling process. In all three cases the correct commanded response values were essentially attained, with only slight changes noted for the control requirements. With regard to the θ-command maneuver, the coupled responses were negligible. Coupling was more pronounced for the other two commands but it was not appreciable. The γ-command maneuver exhibited a coupling of about 0.15 m/sec in speed. The largest amount of coupling was evident in the u-command maneuver where γ approached 0.5°. This coupling would be much more pronounced for a large speed change, such as in high decelerating approaches. As previously noted, however, the coupling effect could be eliminated by retrimming the flight-path angle.

Effect of thrust.- The effect of a change in thrust coefficient on the decoupling process was not investigated. Data in reference 7 show that the effects of changes in thrust on the airplane stability and control derivatives (A and B matrices) are on the same order of magnitude as the effects of α changes; hence, similar effects on the decoupling process are indicated.

Summary of effects.- From the foregoing analysis of the effects of flight-condition changes, use of constant gains in a decoupled control system for certain flight regimes (for example, during approach and landing) appears feasible. It has been shown that changes in the flight conditions do not cause excessive coupling in the response quantities (except for effects on γ during large speed changes) and have only minor effects on the control requirements.

Effect of Airplane Control Lag

The foregoing analysis assumed no control lag; that is, for the step commands, the control surfaces and engine thrust acted instantaneously. In any practical situation, control lag is always present in the flap and horizontal-tail servos as well as in engine thrust with respect to throttle movement. The effects of some typical values of lag are shown in the time histories presented in figures 14 to 17. The values used for ω_n, ζ, τ_1, and τ_3 were the same as those shown in table IV(a).

Method of determining lag effects.- In order to determine the lag effects, lag was incorporated into the system as shown by the A and B matrices in table XI. For thrust lag effect (table XI(a)), δ_{th} was considered as a state variable; that is, the actual (with lag) throttle or thrust control. Hence, the first column of the original B matrix is included in the fifth column of the A matrix. A first-order lag effect was assumed so that

$$\delta'_{th,1} = (\tau_{th}D + 1)\delta'_{th}$$

or

$$\dot{\delta}'_{th} = -\frac{\delta'_{th}}{\tau_{th}} + \frac{\delta'_{th,1}}{\tau_{th}}$$

where D is the operator symbol for time derivative (d/dt), $\delta_{th,1}$ is the input to the servo, and τ_{th} is the thrust response time constant. The above equation appears as the last row in A and B in table XI. A value of 2.0 sec was used for τ_{th}. For decoupling, the same G matrix was used as the one for no lag. The F matrix was the same but a fifth column of zeros was added.

The procedure for investigating the effect of flap lag was the same as that shown for thrust lag. The third column of the original B matrix was included in the fifth column of the A matrix and first order lag was assumed with a time constant of 1.0 sec.

For the horizontal tail, a second-order lag effect was assumed so that

$$K\delta'_{t,1} = (D^2 + D2\zeta_t\omega_t + \omega_t^2)\delta'_t$$

or

$$\ddot{\delta}_t' = -\omega_t^2\delta_t' - 2\zeta_t\omega_t\dot{\delta}_t' + K\delta_{t,1}'$$

where K is merely a proportionality constant relating servo position $\delta_{t,1}$ to control surface position δ_t. The quantities $\dot{\delta}_t$ and $\ddot{\delta}_t$ were considered state variables and were incorporated into the A and B matrices as shown in table XI(b). The second column of the original B matrix is included in the fifth column of the A matrix. As in the case of the other control lags, the original F and G matrices, calculated for no lag, were used in determining the effect of horizontal-tail lag. The F matrix required additional fifth and sixth columns of zeros.

Effect on pitch-command maneuver.- The time histories in figure 14 show the effect of lag in the different controls for a pitch-command maneuver. The horizontal-tail lag has a small time constant, is well damped, and as shown in figure 14(a), has no effect on the decoupling process. (This was also found to be true for the γ- and u-command maneuvers.) The effects of lag in the other two controls are minor for a θ-command maneuver (figs. 14(b) and 14(c)). Except for the initial rates, the control requirements were not appreciably changed and the pitch-command value of 3° was essentially achieved.

Effect of thrust lag.- The effect of thrust lag is shown in figures 15 and 16, respectively, for a γ command and u command. Results are also included for $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$ to show the effect that these quantities have on decoupling with thrust lag present. The servo-input curves, as shown in figure 14, are not included in figures 15 and 16. The response quantities and control requirements for the no-lag condition are shown for comparison when lag is included. There are noticeable differences in the control requirements in all instances. The coupling effects do not appear to be excessive, except for the relatively large Δu of close to -2 m/sec in figure 15(a). In several instances the lag has a large effect on the commanded response, with the most severe effect appearing in figure 16(a) where there is about a 70-percent overshoot in Δu. The γ coupling in figure 16(b) would be much more pronounced in high decelerating approaches but could be corrected by retrimming the flight-path angle.

Effect of flap lag.- The effect of lag in the flap for a γ-command maneuver and u-command maneuver is shown in figures 17(a) and 17(b), respectively. Here again, the flap-servo input curves are not shown. The effects are similar to those shown in figures 15 and 16 for thrust lag, although not as pronounced. (As for the case of thrust lag (fig. 16(b)), large flap-lag effects on γ coupling would have to be trimmed out during high decelerating approaches.) Most notably, the changes in the control requirements are small, except for the initial rates. The flap rate in figure 17(a) of about 30°/sec during the γ-command maneuver (with lag) is well above the 5°/sec limit (table I); however, it applies to the extreme case of an instantaneous step-command input. Also, the correct command responses were attained in figures 17(a) and 17(b). It is of interest to note that the horizontal tail essentially takes over for the deficiency in the flap.

Summary of effects.- The effects of control lag on the decoupling process were more pronounced than the effects of other error sources investigated. The magnitude of the effects varied for the different controls and depended on the type of command maneuver. Pitch-command maneuvers were essentially unaffected by lag in any of the controls. Also, the effect of horizontal-tail lag was negligible for the different command maneuvers. Thrust-lag effect was the most critical with regard to coupling of responses and to attaining a desired command response. Except for the initial rates, the effects on the control requirements were not excessive. The coupling effects were generally moderate, with about 0.5° in γ and θ, about 0.5 m/sec in u. Coupling in γ would be much more pronounced for larger speed changes, but could be corrected by retrimming the flight-path angle.

Inclusion of Lag in Decoupled Design

The foregoing analysis clearly indicates that control lag could seriously affect decoupling. However, lag can be taken into account in the decoupling design, provided the lag characteristics of the controls are known. The procedure used for including a first-order lag effect in the design of a decoupled control system is presented in appendix B. Example time histories with the control lag included are shown in figures 18 to 20 for the three different command maneuvers. Perfect decoupling was achieved in all cases. In all cases the horizontal-tail servo time constant τ_t and the thrust time constant τ_{th} were assumed to be 0.2 sec and 2.0 sec, respectively. Two different values were used for the flap servo time constant, 0.2 and 1.0 sec. Different values were used for the airplane dynamic response constants, as indicated in the figure titles. These airplane constants are noted only for the commanded response quantity inasmuch as the constants for the other response quantities were irrelevant because these quantities were held to zero.

Figure 18 shows examples of a γ-command maneuver for the two types of airplane dynamic response characteristics described in appendix B. (See eqs. (B14) and (B16).) As can be noted from appendix B, the results in figure 18, as well as in figures 19 and 20, correspond to the case where $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$. The results in figure 18(a) are shown for two different values of flap servo time constant τ_f. The only difference in the time histories is in the flap servo input command. A good response in γ is obtained, but it is obtained at the expense of an excessive initial flap rate of about 40°/sec. (See table I.) The high value of the throttle command is relatively insignificant because the actual response value is very small. In figure 18(b), a much lower initial flap rate (about 10°/sec) is required; however, the γ response is much slower because of the change in the airplane dynamic response characteristics. Even though the flap rate exceeds the limit of 5°/sec, it should be remembered that these time histories represent the extreme case of instantaneous step-command inputs.

An example of a pitch-command maneuver is presented in figure 19 for the airplane pitch-response characteristics indicated in the title. The dynamic characteristics of the combined airplane- and control-response lags, as

described in appendix B, are included in the figure. The results show that an adequate pitch response is obtained. A high initial horizontal-tail rate of about 30°/sec is required; this is well within the limit value (table I).

Figure 20 presents various examples of a u-command maneuver. The different combinations used for the control- and speed-dynamic response characteristics are described in appendix B. (See eqs. (B14) and (B16).) All the cases in figure 20 show that the actual response values of the flap and horizontal tail remain relatively close to the command values, whereas the opposite is true for the throttle owing to its higher τ of 2.0 sec. Because the throttle is the primary control for speed in the decoupled design, this result indicates that the speed response ordinarily is affected appreciably if thrust lag is taken into account in the decoupled control system design. The τ_f of 0.2 sec used for the flap is unrealistically small; however, increasing it to larger values would affect only the flap-command value.

Figure 20(a) shows that a good speed response is obtained for the dynamic characteristics that were assumed. This response, however, was obtained at the expense of an excessive initial flap rate of 40°/sec.

In comparison to figure 20(a), figures 20(b), (c), and (d) show slower speed responses; however, the control rates are more reasonable. In figure 20(b), the speed response is not adequate; the actual throttle response value fails to reach the command value. In figure 20(c), the speed response time constant τ_1 has been reduced to 0.5 sec. The speed response is better, but still too slow. The dynamic characteristics calculated for the combinations of τ_1 and τ_{th} are equivalent to the second-order system shown in the subtitle. Figure 20(d) is the same as figure 20(c), except that the damping ratio has been reduced by one-half. It can be seen that the speed response is greatly improved; however, about twice as much flap deflection is required for the maneuver.

Summary of lag analysis results.- The foregoing results give only a brief look at the design possibilities which incorporate control lag in the decoupled system. Only one value for thrust lag and one value for horizontal-tail control lag were investigated together with several different airplane dynamic responses. There are countless combinations of control and airplane response characteristics that could be applied in the basic design of a decoupled system. The responses would necessarily have to be selected to match the control limits.

The preliminary results presented in this paper indicate that, with proper selection of the airplane dynamic characteristics and appropriate adjustment of the control-lag characteristics, a decoupled control system can be designed to account for the adverse effects of control lag. For the θ- and γ-command cases, there appeared to be no problem in attaining adequate command response and reasonable control requirements. The u-command maneuver was the most critical and required careful selection of the airplane dynamic characteristics.

CONCLUSIONS

A study has been conducted to determine the effect of various error sources on a decoupled control system. The analysis was performed in connection with the longitudinal-control system on a simulated externally blown jet-flap STOL aircraft. The system employed the throttle, horizontal tail, and flaps to decouple the forward velocity, pitch angle, and flight-path angle.

It was shown that, despite the large errors employed in the study, the effects of the various errors on the decoupling process were generally minor in nature. Specific conclusions reached in regard to each of the error sources were:

1. Effects of imperfect knowledge of the airplane characteristics (elements of the A and B matrices) were generally small. Decoupling was completely lost in only a few cases and this loss applied to a 50- to 100-percent error in the corresponding matrix element. As contrasted to B-matrix errors, the effects of A-matrix errors on decoupling as well as on the control requirements were essentially linearly related to the magnitude of the errors. This result indicates that designing adequate decoupled control systems requires better knowledge of the B matrix than of the A matrix.

2. The dominant effect of state-variable measurement error occurred in the command response quantity in which the error existed; the control requirements and coupling in the other response quantities were not materially affected. The effect of measurement error was found to be nonlinear with respect to the magnitude of the error and (in the case of flight-path angle and speed) with respect to the nominal value of the pertinent airplane dynamic response characteristic defined in the A matrix. Because the error in the flight-path angle and speed-command response quantities depends on the magnitude of the nominal value of the A-matrix element, this error could be reduced if the A matrix were changed by altering the aircraft aerodynamic design.

In general, the effect of measurement error on the command response would not be critical in a decoupled control system. The pilot would normally recognize this effect on the response he is commanding and could compensate for this error with an additional command input; only minor additional effects on the decoupling process would result.

3. Except for flight-path angle coupling during large speed changes (for example, high decelerating approaches), changes in the flight conditions did not cause excessive coupling in the response quantities and had only minor effects on the control requirements. Thus, use of constant gains in a decoupled control system for certain flight regimes appears feasible; consequently, the need for an onboard computer is avoided. The major effect of flight-condition changes occurred in the commanded response quantity; this effect, however, could be recognized by the pilot and could be easily controlled with an additional command input.

4. The effects of control lag on the decoupling process were more pronounced than the effects of other error sources. Thrust-lag effect was the most critical with regard to coupled responses and with regard to attaining a

desired speed-command response, especially during high decelerating approaches. Control-lag effects on the control requirements were not excessive, except for the flap during flight-path-angle command maneuvers.

5. The excessive flight-path-angle coupling caused by control lag and by changes in flight condition during speed-command maneuvers could be eliminated by retrimming the flight-path angle, either automatically or by the pilot.

6. Provided the lag characteristics of the controls are known, lag could be incorporated in the design of the decoupled system. Such incorporation would permit complete decoupling; however, proper selection of the airplane dynamic characteristics and appropriate adjustments in the control-lag characteristics would be required. For pitch-command maneuvers there appeared to be no problem in attaining adequate command response and reasonable control requirements. The speed-command maneuver was the most critical and would normally require careful selection of the airplane dynamic characteristics.

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APPENDIX A

DECOUPLED LONGITUDINAL CONTROLS

The three longitudinal equations of motion were linearized as perturbations about an equilibrium condition in equations (1-59) of reference 13. These three equations can be solved simultaneously to give

$$\frac{d^2\theta'}{dt^2} = \frac{u_0^2}{2\mu K_Y^2 \bar{c}^2}\left[\frac{\bar{c}}{u_0}\left(\frac{C_{m_q} + C_{m_{\dot{\alpha}}}}{2}\right)\frac{d\theta'}{dt} + \left(C_{m_\alpha} + \frac{C_{m_{\dot{\alpha}}}C_{z_\alpha}}{4\mu}\right)\alpha' + \left(C_{m_u} + \frac{C_{m_{\dot{\alpha}}}C_{z_u}}{4\mu}\right)u'\right.$$

$$\left. + \left(C_{m_{\delta_{th}}} + \frac{C_{m_{\dot{\alpha}}}C_{z_{\delta_{th}}}}{4\mu}\right)\delta'_{th} + \left(C_{m_{\delta_t}} + \frac{C_{m_{\dot{\alpha}}}C_{z_{\delta_t}}}{4\mu}\right)\delta'_t + \left(C_{m_{\delta_f}} + \frac{C_{m_{\dot{\alpha}}}C_{z_{\delta_f}}}{4\mu}\right)\delta'_f\right] \tag{A1}$$

$$\frac{d\alpha'}{dt} = \frac{u_0}{2\mu\bar{c}}\left(2\mu\,\frac{\bar{c}}{u_0}\,\frac{d\theta'}{dt} + C_{z_\alpha}\alpha' + C_{z_u}u' + C_{z_{\delta_{th}}}\delta'_{th} + C_{z_{\delta_t}}\delta'_t + C_{z_{\delta_f}}\delta'_f\right) \tag{A2}$$

$$\frac{du'}{dt} = \frac{u_0}{2\mu\bar{c}}\left[C_W\theta' + \frac{\bar{c}}{u_0}\left(\frac{C_{X_q} + C_{X_{\dot{\alpha}}}}{2}\right)\frac{d\theta'}{dt} + \left(C_{X_\alpha} + \frac{C_{X_{\dot{\alpha}}}C_{z_\alpha}}{4\mu}\right)\alpha' + \left(C_{X_u} + \frac{C_{X_{\dot{\alpha}}}C_{z_u}}{4\mu}\right)u'\right.$$

$$\left. + \left(C_{X_{\delta_{th}}} + \frac{C_{X_{\dot{\alpha}}}C_{z_{\delta_{th}}}}{4\mu}\right)\delta'_{th} + \left(C_{X_{\delta_t}} + \frac{C_{X_{\dot{\alpha}}}C_{z_{\delta_t}}}{4\mu}\right)\delta'_t + \left(C_{X_{\delta_f}} + \frac{C_{X_{\dot{\alpha}}}C_{z_{\delta_f}}}{4\mu}\right)\delta'_f\right] \tag{A3}$$

The terms $C_{z_{\dot{\alpha}}}$ and C_{z_q} given in reference 13 were neglected. Also, $\sin\Theta$ was assumed to equal 0 and $\cos\Theta$ to equal 1 (Θ is the angle between the horizon and X-equilibrium axis). The primed parameters in equations (A1), (A2), and (A3) are perturbations from the equilibrium or trim conditions of the airplane in nondimensional form; that is,

$$\theta' = \theta - \theta_0 \tag{A4}$$

$$\alpha' = \alpha - \alpha_0 \tag{A5}$$

$$u' = \frac{u - u_0}{u_0} \tag{A6}$$

and where

$$\mu = \frac{m}{\rho S \bar{c}} \tag{A7}$$

$$K_Y^2 = \frac{I_Y}{m\bar{c}^2} \tag{A8}$$

The weight and dimensional characteristics of the simulated airplane are presented in table I. The values used for the basic aerodynamic coefficients are given in table II. Constant coefficients were employed in the linearized longitudinal equations of motion corresponding to an angle of attack of 10°, a forward velocity of 30.48 m/sec, and a thrust coefficient of 1.87.

The linearized longitudinal equations of motion can be written in state vector notation (after adding the fourth equation $\dot{\theta} = q$) as

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} \tag{A9}$$

where the state vector is

$$\bar{x} = \begin{pmatrix} \theta' \\ \dot{\theta}' \\ \alpha' \\ u' \end{pmatrix} \tag{A10}$$

and the control vector is

$$\bar{u} = \begin{pmatrix} \delta'_{th} \\ \delta'_t \\ \delta'_f \end{pmatrix} \tag{A11}$$

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The airplane has a spoiler control, but it was not used in the study and, therefore, was not included in equations (A1) to (A3). The control law used is given as

$$\bar{u} = F\bar{x} + G\bar{v} \tag{A12}$$

where $\bar{v}$ is the vector of commanded pilot inputs u_c, θ_c, and γ_c that are to be controlled in a decoupled manner. The output equation

$$\bar{y} = C\bar{x} \tag{A13}$$

determines which state outputs are to be decoupled.

Substitution of equation (A12) into equation (A9) yields the augmented equation

$$\dot{\bar{x}} = (A + BF)\,\bar{x} + BG\bar{v} \tag{A14}$$

The Laplace transform of the result can be written as

$$s\hat{\bar{x}}(s) - \bar{x}(0) = (A + BF)\,\hat{\bar{x}}(s) + BG\hat{\bar{v}}(s) \tag{A15}$$

where the circumflex indicates the Laplace transform. Equation (A15) can be written as

$$(sI - A - BF)\,\hat{\bar{x}}(s) = \bar{x}(0) + BG\hat{\bar{v}}(s) \tag{A16}$$

Substituting the Laplace transform of equation (A13) into equation (A16) gives the transfer function between $\bar{y}$ and $\bar{v}$ as

$$H = C(sI - A - BF)^{-1}BG \tag{A17}$$

For a completely decoupled system, the matrices F and G must be chosen so that the transfer function matrix H is diagonal and nonsingular. The computer program reported in reference 9 can be used to compute the desired F and G matrices.

APPENDIX A

Although the Gilbert program of reference 9 is based on a sophisticated algebraic theory of linear systems, the derivation of the decoupling control law (eq. (A12)) can be simply explained by the method of Rekasius (ref. 1). The method is based on time differentiating each y-element until control quantities $\bar{u}$ appear explicitly in the expression for y; the associated v is then defined to give desirable transfer functions from v to y and to permit solving for the decoupling control law in the form of equation (A12). For purposes of clarity, the primes are dropped from the response and control variables.

For speed control only, let

$$y_1 = u \; ; \; \dot{y}_1 = \dot{u} \tag{A18}$$

From equation (A3) and table III, then

$$\dot{y}_1 = a_{4,1}\theta + a_{4,3}\alpha + a_{4,4}u + b_{4,1}\delta_{th} + b_{4,2}\delta_t + b_{4,3}\delta_f \tag{A19}$$

(Note that eq. (A19) contains required control quantities.) Adding first-order dynamics yields

$$\dot{y}_1 + \frac{u}{\tau_1} = a_{4,1}\theta + a_{4,3}\alpha + \left(a_{4,4} + \frac{1}{\tau_1}\right)u + b_{4,1}\delta_{th} + b_{4,2}\delta_t + b_{4,3}\delta_f = v_1 \tag{A20}$$

Similarly, for pitch control only, let

$$y_2 = \theta \; ; \; \dot{y}_2 = \dot{\theta} \; ; \; \ddot{y}_2 = \ddot{\theta} \tag{A21}$$

From equation (A1) and table III, then

$$\ddot{y}_2 = a_{2,2}q + a_{2,3}\alpha + a_{2,4}u + b_{2,1}\delta_{th} + b_{2,2}\delta_t + b_{2,3}\delta_f \tag{A22}$$

(Eq. (A22) contains control quantities.) Adding second-order dynamics yields

$$\ddot{y}_2 + 2\zeta\omega_n q + \omega_n^2\theta = \omega_n^2\theta + (a_{2,2} + 2\zeta\omega_n)q + a_{2,3}\alpha + a_{2,4}u + b_{2,1}\delta_{th} + b_{2,2}\delta_t + b_{2,3}\delta_f = v_2 \tag{A23}$$

Finally, for flight-path-angle control only, let

$$y_3 = \theta - \alpha \; ; \; \dot{y}_3 = \dot{\theta} - \dot{\alpha} \tag{A24}$$

From equation (A2) and table III (and noting that $\dot{\theta} = q$), then

$$\dot{y}_3 = (1 - a_{3,2})q - a_{3,3}\alpha - a_{3,4}u - b_{3,1}\delta_{th} - b_{3,2}\delta_t - b_{3,3}\delta_f \tag{A25}$$

(Eq. (A25) contains control quantities.) Adding first-order dynamics yields

$$\dot{y}_3 + \frac{\alpha}{\tau_3} = (1 - a_{3,2})q - \left(a_{3,3} - \frac{1}{\tau_3}\right)\alpha - a_{3,4}u - b_{3,1}\delta_{th} - b_{3,2}\delta_t - b_{3,3}\delta_f = v_3 \tag{A26}$$

Equations (A20), (A23), and (A26) can be collected to form

$$\left.\begin{aligned} v_1 &= a_{4,1}\theta + 0q + a_{4,3}\alpha + \left(a_{4,4} + \frac{1}{\tau_1}\right)u + b_{4,1}\delta_{th} + b_{4,2}\delta_t + b_{4,3}\delta_f \\ v_2 &= \omega_n^2\theta + (a_{2,2} + 2\zeta\omega_n)q + a_{2,3}\alpha + a_{2,4}u + b_{2,1}\delta_{th} + b_{2,2}\delta_t + b_{2,3}\delta_f \\ v_3 &= 0\theta + (1 - a_{3,2})q - \left(a_{3,3} - \frac{1}{\tau_3}\right)\alpha - a_{3,4}u - b_{3,1}\delta_{th} - b_{3,2}\delta_t - b_{3,3}\delta_f \end{aligned}\right\} \tag{A27}$$

Equations (A27) can be written in matrix form as

$$\bar{v} = F'\bar{x} + G'\bar{u} \tag{A28}$$

or

$$\bar{u} = -[G']^{-1}F'\bar{x} + [G']^{-1}\bar{v} \tag{A29}$$

APPENDIX A

Providing G' is not singular, then

$$\bar{u} \triangleq F\bar{x} + G\bar{v} \tag{A30}$$

where $F = -[G']^{-1}F'$ and $G = [G']^{-1}$ are the feedback and feedforward gain matrices required for decoupling. The values of τ_1, τ_3, ζ, and ω_n are free to be selected for any desired response.

APPENDIX B

DECOUPLED LONGITUDINAL CONTROLS WITH LAG

In this section the longitudinal equations of motion given in appendix A are altered in order to take control lag into account in the decoupling design process by defining the control vector $\bar{u}$ as

$$\bar{u} = \begin{pmatrix} \delta'_{th,1} \\ \delta'_{t,1} \\ \delta'_{f,1} \end{pmatrix} \tag{B1}$$

where the subscript 1 refers to servo control. In the design procedure, lags are considered first order by letting

$$(\tau_{th}D + 1)\delta'_{th} = \delta'_{th,1} \quad \text{or} \quad \dot{\delta}'_{th} = -\frac{\delta'_{th}}{\tau_{th}} + \frac{\delta'_{th,1}}{\tau_{th}} \tag{B2}$$

$$(\tau_{t}D + 1)\delta'_{t} = \delta'_{t,1} \quad \text{or} \quad \dot{\delta}'_{t} = -\frac{\delta'_{t}}{\tau_{t}} + \frac{\delta'_{t,1}}{\tau_{t}} \tag{B3}$$

$$(\tau_{f}D + 1)\delta'_{f} = \delta'_{f,1} \quad \text{or} \quad \dot{\delta}'_{f} = -\frac{\delta'_{f}}{\tau_{f}} + \frac{\delta'_{f,1}}{\tau_{f}} \tag{B4}$$

where D is the operator symbol for the derivative with respect to time and δ'_{th}, δ'_{t}, and δ'_{f} become state variables.

Equations (B2) to (B4), combined with the longitudinal equations of motion of appendix A can be written in matrix form as

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} \tag{B5}$$

where

$$\dot{\bar{x}} = \begin{pmatrix} \dot{\theta}' \\ \dot{q}' \\ \dot{\alpha}' \\ \dot{u}' \\ \dot{\delta}'_{th} \\ \dot{\delta}'_{t} \\ \dot{\delta}'_{f} \end{pmatrix} \tag{B6}$$

Matrix A in terms of values listed in table III(a) is

$$A = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_{th} \\ \dot{\delta}_{t} \\ \dot{\delta}_{f} \end{array} \begin{array}{c} \begin{array}{ccccccc} \theta & q & \alpha & u & \delta_{th} & \delta_{t} & \delta_{f} \end{array} \\ \begin{bmatrix} 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.2300 & -0.5200 & 0.2250 & -0.1460 & -2.380 & 0.1487 \\ 0 & 1.0 & -0.3680 & -0.6400 & -0.1850 & -0.0676 & -0.1712 \\ -0.3195 & 0 & 0.1570 & -0.1018 & 0.1047 & -0.01406 & -0.1190 \\ 0 & 0 & 0 & 0 & -1/\tau_{th} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/\tau_{t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau_{f} \end{bmatrix} \end{array} \tag{B7}$$

and matrix B is

$$B = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_{th} \\ \dot{\delta}_{t} \\ \dot{\delta}_{f} \end{array} \begin{array}{c} \begin{array}{ccc} \delta_{th,1} & \delta_{t,1} & \delta_{f,1} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/\tau_{th} & 0 & 0 \\ 0 & 1/\tau_{t} & 0 \\ 0 & 0 & 1/\tau_{f} \end{bmatrix} \end{array} \tag{B8}$$

(In the A matrix, the $C_{z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$ case is used.)

The procedure for determining the F and G matrices required for decoupling is similar to that shown in appendix A. There is a difference in the required number of differentiations of y, as shown below. Here again, the primes are dropped from the response and control variables for purposes of clarity.

For speed control only, let

$$y_1 = u \; ; \; \dot{y}_1 = \dot{u} \tag{B9}$$

thus, from equations (B7) and (B8)

$$\dot{y}_1 = a_{4,1}\theta + a_{4,3}\alpha + a_{4,4}u + a_{4,5}\delta_{th} + a_{4,6}\delta_{t} + a_{4,7}\delta_{f} \tag{B10}$$

It is evident, however, that equation (B10) does not contain a servo-control variable which is required. Therefore y_1 must be differentiated again

$$\ddot{y}_1 = \ddot{u} \tag{B11}$$

so that

$$\ddot{y}_1 = a_{4,1}\dot{\theta} + a_{4,3}\dot{\alpha} + a_{4,4}\dot{u} + a_{4,5}\dot{\delta}_{th} + a_{4,6}\dot{\delta}_{t} + a_{4,7}\dot{\delta}_{f} \tag{B12}$$

APPENDIX B

Substituting for the differentials

$$\ddot{y}_1 = a_{4,1}(a_{1,2}q) + a_{4,3}(a_{3,2}q + a_{3,3}\alpha + a_{3,4}u + a_{3,5}\delta_{th} + a_{3,6}\delta_t + a_{3,7}\delta_f)$$

$$+ a_{4,4}(a_{4,1}\theta + a_{4,3}\alpha + a_{4,4}u + a_{4,5}\delta_{th} + a_{4,6}\delta_t + a_{4,7}\delta_f)$$

$$+ a_{4,5}\left(-\frac{1}{\tau_{th}}\delta_{th} + \frac{1}{\tau_{th}}\delta_{th,1}\right) + a_{4,6}\left(-\frac{1}{\tau_t}\delta_t + \frac{1}{\tau_t}\delta_{t,1}\right)$$

$$+ a_{4,7}\left(-\frac{1}{\tau_f}\delta_f + \frac{1}{\tau_f}\delta_{f,1}\right) \qquad \text{(B13)}$$

(Eq. (B13) contains a servo-control variable which is required.)

As for including dynamics in equation (B13), the results indicate second-order dynamics for the u response; therefore, the u response is modeled as

$$\left(s + \frac{1}{\tau_1}\right)\left(s + \frac{1}{\tau_{th}}\right) \quad \text{or} \quad s^2 + s\left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right) + \frac{1}{\tau_1\tau_{th}} \qquad \text{(B14)}$$

The speed response would then have the engine response lag superimposed on the airplane response lag. Adding the dynamic response to equation (B13) yields

$$\ddot{u} + \dot{u}\left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right) + u\left(\frac{1}{\tau_1\tau_{th}}\right) = v_1$$

$$= \theta\left[a_{4,4} + \left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right)\right]a_{4,1} + q(a_{4,1} + a_{4,3})$$

$$+ \alpha\left[a_{3,3} + a_{4,4} + \left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right)\right]a_{4,3}$$

$$+ u\left[(a_{4,4})^2 + a_{4,3}a_{3,4} + \frac{1}{\tau_1\tau_{th}} + a_{4,4}\left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right)\right]$$

(Equation (B15) continued on next page)

$$+ \delta_{th}\left\{a_{4,3}a_{3,5} + a_{4,5}\left[a_{4,4} - \frac{1}{\tau_{th}} + \left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right)\right]\right\}$$

$$+ \delta_{t}\left\{a_{4,3}a_{3,6} + a_{4,6}\left[a_{4,4} - \frac{1}{\tau_{t}} + \left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right)\right]\right\}$$

$$+ \delta_{f}\left\{a_{4,3}a_{3,7} + a_{4,7}\left[a_{4,4} - \frac{1}{\tau_{f}} + \left(\frac{1}{\tau_1} + \frac{1}{\tau_{th}}\right)\right]\right\}$$

$$+ \delta_{th,1}\left(a_{4,5}\frac{1}{\tau_{th}}\right) + \delta_{t,1}\left(a_{4,6}\frac{1}{\tau_{t}}\right) + \delta_{f,1}\left(a_{4,7}\frac{1}{\tau_{f}}\right) \qquad \text{(B15)}$$

Instead of using the expression given by equation (B14), the second-order dynamics for the u response could be represented by

$$\ddot{u} + 2\zeta_1\omega_1\dot{u} + \omega_1^2 u \qquad \text{(B16)}$$

and the terms $\left(\frac{1}{\tau} + \frac{1}{\tau_{th}}\right)$ and $\frac{1}{\tau_1\tau_{th}}$ in equation (B15) would be replaced by $(2\zeta_1\omega_1)$ and ω_1^2, respectively.

Similarly, for pitch control only, let

$$\dddot{y}_2 = \ddot{q} \qquad \text{(B17)}$$

so that, from equations (B7) and (B8)

$$\dddot{y}_2 = a_{2,2}\dot{q} + a_{2,3}\dot{\alpha} + a_{2,4}\dot{u} + a_{2,5}\dot{\delta}_{th} + a_{2,6}\dot{\delta}_{t} + a_{2,7}\dot{\delta}_{f}$$

APPENDIX B

Substituting for the differentials,

$$\dddot{y}_2 = a_{2,2}(a_{2,2}q + a_{2,3}\alpha + a_{2,4}u + a_{2,5}\delta_{th} + a_{2,6}\delta_t + a_{2,7}\delta_f)$$

$$+ a_{2,3}(a_{3,2}q + a_{3,3}\alpha + a_{3,4}u + a_{3,5}\delta_{th} + a_{3,6}\delta_t + a_{3,7}\delta_f)$$

$$+ a_{2,4}(a_{4,1}\theta + a_{4,3}\alpha + a_{4,4}u + a_{4,5}\delta_{th} + a_{4,6}\delta_t + a_{4,7}\delta_f)$$

$$+ a_{2,5}\left(-\frac{1}{\tau_{th}}\delta_{th} + \frac{1}{\tau_{th}}\delta_{th,1}\right) + a_{2,6}\left(-\frac{1}{\tau_t}\delta_t + \frac{1}{\tau_t}\delta_{t,1}\right)$$

$$+ a_{2,7}\left(-\frac{1}{\tau_f}\delta_f + \frac{1}{\tau_f}\delta_{f,1}\right) \qquad \text{(B18)}$$

(Eq. (B18) contains servo-control variables.)

As for including dynamics in equation (B18), the results indicate third-order dynamics for the pitch response; therefore, it appears logical to assume for the response

$$\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)\left(s + \frac{1}{\tau_t}\right) \qquad \text{(B19)}$$

The lag in the horizontal tail would then be superimposed on the airplane response lag. Adding the dynamic response to equation (B18) yields

$$\ddot{q} + \dot{q}\left(2\zeta\omega_n + \frac{1}{\tau_t}\right) + q\left(\omega_n^2 + \frac{1}{\tau_t}\,2\zeta\omega_n\right) + \theta\left(\frac{1}{\tau_t}\,\omega_n^2\right) = v_2 \qquad \text{(B20)}$$

or

$$\ddot{q} + \dot{q}P + qQ + \theta R = v_2 = \theta(a_{2,4}a_{4,1} + R) + q\left[(a_{2,2})^2 + a_{2,3}a_{3,2} + Q + a_{2,2}P\right]$$

$$+ \alpha\left[a_{2,3}(a_{2,2} + a_{3,3} + P) + a_{2,4}a_{4,3}\right]$$

(Equation (B21) continued on next page)

$$+ u\left[a_{2,4}(a_{2,2} + a_{4,4} + P) + a_{2,3}a_{3,4}\right]$$

$$+ \delta_{th}\left[a_{2,3}a_{3,5} + a_{2,4}a_{4,5} + a_{2,5}\left(a_{2,2} - \frac{1}{\tau_{th}} + P\right)\right]$$

$$+ \delta_{t}\left[a_{2,3}a_{3,6} + a_{2,4}a_{4,6} + a_{2,6}\left(a_{2,2} - \frac{1}{\tau_{t}} + P\right)\right]$$

$$+ \delta_{f}\left[a_{2,3}a_{3,7} + a_{2,4}a_{4,7} + a_{2,7}\left(a_{2,2} - \frac{1}{\tau_{f}} + P\right)\right]$$

$$+ \delta_{th,1}\left(a_{2,5}\,\frac{1}{\tau_{th}}\right) + \delta_{t,1}\left(a_{2,6}\,\frac{1}{\tau_{t}}\right) + \delta_{f,1}\left(a_{2,7}\,\frac{1}{\tau_{f}}\right) \qquad \text{(B21)}$$

where

$$P = 2\zeta\omega_n + \frac{1}{\tau_t}$$

$$Q = \omega_n^2 + \frac{1}{\tau_t}\,2\zeta\omega_n$$

$$R = \frac{1}{\tau_t}\,\omega_n^2$$

The equation for v_3, representing γ control only, can be obtained by letting $y_3 = \theta - \alpha$ and following the procedure for v_1. (See eqs. (B9) to (B15).) The equations for v_1, v_2, and v_3 can be combined in like manner to equation (A27) and solved for the gain matrices F and G as in equation (A30).

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TABLE I.- WEIGHT AND DIMENSIONAL CHARACTERISTICS OF SIMULATED AIRCRAFT

Weight, N	245 096
Wing area, m^2	78
Wing span, m	24
Mean aerodynamic chord, m	3.58
Center-of-gravity location, percent $\bar{c}$	40
I_Y, kg-m^2	334 637
Maximum control-surface deflections:	
δ_t, deg	±10
δ_f, deg	0 to 90
Maximum control-surface deflection rates:	
$\dot{\delta}_t$, deg/sec	50
$\dot{\delta}_f$, deg/sec	5

TABLE II.- BASIC AERODYNAMIC CHARACTERISTICS USED IN STUDY

[Values in parentheses used for extra cases]

Parameter	Value for -	
	$\alpha = 10^{\circ}$	$\alpha = 5^{\circ}$
$C_W (\sim C_Z)$	-5.71	-5.10
C_{Z_α}	-6.567	-7.20
C_{X_α}	2.81	1.97
C_{m_α}	-1.424	-1.80
$C_{X_{\dot{\alpha}}}$	0	0
$C_{m_{\dot{\alpha}}}$	-13.14	-13.70
C_{X_q}	0	0
C_{Z_u}	-11.42	-10.20
C_{X_u}	-1.82	-1.40
C_{m_u}	0	0
C_{m_q}	-32.86	-34.30
$C_{Z_{\delta_t}}$	-1.209	-1.157
$C_{X_{\delta_t}}$	-.252	-.0688
$C_{m_{\delta_t}}$	-5.272	-5.56
$C_{Z_{\delta_{th}}}$	0, (-3.3)	0
$C_{m_{\delta_{th}}}$	0, (-.468)	0
$C_{X_{\delta_{th}}}$	1.87	1.87
$C_{Z_{\delta_f}}$	-3.06	-3.40
$C_{X_{\delta_f}}$	-2.126	-2.225
$C_{m_{\delta_f}}$	.195	.126

TABLE III.- VALUES OF COEFFICIENTS FOR A AND B MATRICES USED IN STUDY

[Values in parentheses used for extra cases]

(a) $\alpha = 10^{\circ}$

$$A = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \end{array} \begin{array}{c} \begin{array}{cccc} \theta & q & \alpha & u \end{array} \\ \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 0 & -1.23 & -0.52 & 0.225 \\ 0 & 1.0 & -0.368 & -0.64 \\ -0.3195 & 0 & 0.157 & -0.1018 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Throttle} & \text{Horizontal-tail} & \text{Flap} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 \\ (-0.146) & -2.38 & 0.1487 \\ 0 \\ (-0.185) & -0.0676 & -0.1712 \\ 0.1047 & -0.01406 & -0.1190 \end{bmatrix} \end{array}$$

(b) $\alpha = 5^{\circ}$

$$A = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \end{array} \begin{array}{c} \begin{array}{cccc} \theta & q & \alpha & u \end{array} \\ \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 0 & -1.285 & -0.6725 & 0.209 \\ 0 & 1.0 & -0.403 & -0.57 \\ -0.286 & 0 & 0.1105 & -0.0785 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Throttle} & \text{Horizontal-tail} & \text{Flap} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2.515 & 0.127 \\ 0 & -0.0646 & -0.19 \\ 0.1047 & -0.00383 & -0.1247 \end{bmatrix} \end{array}$$

TABLE IV.- TYPICAL GAINS FOR DECOUPLING LONGITUDINAL AIRPLANE MOTIONS u, θ, AND γ

(a) Standard case

$\omega_n = 2$ rad/sec $\quad \tau_1 = 1.0$ sec
$\zeta = 0.7 \quad \tau_3 = 1.0$ sec

$$F = \begin{bmatrix} -3.99115 & -0.20247 & 2.69253 & -12.78503 \\ 1.28404 & 0.64378 & 0.01187 & -0.13568 \\ -6.34813 & -0.25420 & 3.68691 & -3.68474 \end{bmatrix}$$

$$G = \begin{bmatrix} 9.55110 & 0.12896 & 6.52689 \\ 0 & -0.41005 & 0.35616 \\ 0 & 0.16191 & 5.70049 \end{bmatrix}$$

(b) $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$ case

$\omega_n = 2$ rad/sec $\quad \tau_1 = 1.0$ sec
$\zeta = 0.7 \quad \tau_3 = 1.0$ sec

$$F = \begin{bmatrix} -1.82356 & -0.09250 & 1.23023 & -5.84152 \\ 1.51337 & 0.65541 & -0.14285 & 0.59894 \\ -4.46814 & -0.15885 & 2.41860 & 2.33758 \end{bmatrix}$$

$$G = \begin{bmatrix} 4.36393 & 0.05892 & 2.98216 \\ -0.54880 & -0.41746 & -0.01887 \\ -4.49900 & 0.10117 & 2.62603 \end{bmatrix}$$

TABLE IV.- Concluded

(c) Case with lag included in the decoupling design

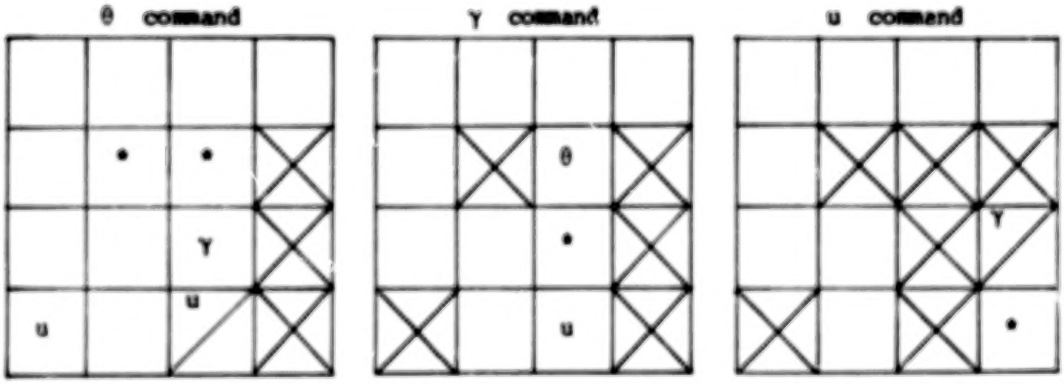
ω_n = 2 rad/sec τ_t = 0.2 sec
ζ = 0.7 τ_f = 1.0 sec
τ_1 = 0.5 sec τ_{th} = 2.0 sec
τ_3 = 1.0 sec

$$F = \begin{bmatrix} 0.22497 & -0.23249 & 0.76821 & 1.58269 & -0.30732 & -0.04432 & 0.05331 \\ -11.77187 & -0.62304 & -1.88429 & -0.40576 & 1.62150 & -3.03945 & 1.39079 \\ 0.19593 & 2.54177 & -2.64842 & -7.55244 & 1.11315 & 0.11508 & -1.04370 \end{bmatrix}$$

$$G = \begin{bmatrix} -0.10976 & -0.08349 & -0.00377 \\ 8.72786 & 0.11785 & 5.96432 \\ -4.49900 & 0.10117 & 2.62603 \end{bmatrix}$$

TABLE V.- EFFECT OF UNCERTAINTY IN A-MATRIX COEFFICIENTS ON DECOUPLING

	θ	$\dot{\theta}$	α	u
q	0	1	0	0
$\ddot{\theta}$	0	C_{m_q} $C_{m_{\dot{\alpha}}}$	C_{m_α} $C_{m_{\dot{\alpha}}}$ C_{Z_α}	$C_{m_{\dot{u}}}$ C_{Z_u}
$\dot{\alpha}$	0	1	C_{Z_α}	C_{Z_u}
$\dot{u}$	C_W	0	C_{X_α}	C_{X_u}



•θ overshoot, 20 percent.

•γ overshoot, 50 percent.

•u overshoot, 10 percent.

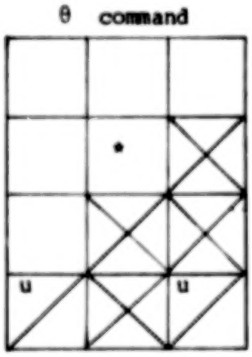
Key:

- Value cannot change
- No significant effect on decoupling
- θ γ u — Decoupling lost for θ, γ, and u, respectively
- θ γ u — Partial decoupling lost for θ, γ, and u, respectively
- • — The terms "overshoot" and "undershoot" refer to steady-state error.

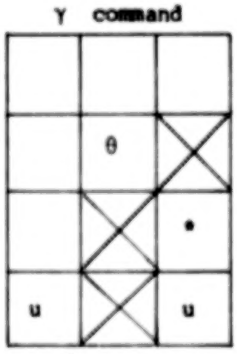
TABLE VI.- EFFECT OF UNCERTAINTY IN B-MATRIX COEFFICIENTS ON DECOUPLING

	δ_{th}	δ_t	δ_f
q	0	0	0
$\ddot{\theta}$		$C_{m\delta_t}$ *$C_{m\dot{\alpha}}$ *$C_{z\delta_t}$	$C_{m\delta_f}$ $C_{m\dot{\alpha}}$ $C_{z\delta_f}$
$\dot{\alpha}$		$C_{z\delta_t}$	$C_{z\delta_f}$
$\dot{u}$	$C_{x\delta_{th}}$	$C_{x\delta_t}$	$C_{x\delta_f}$

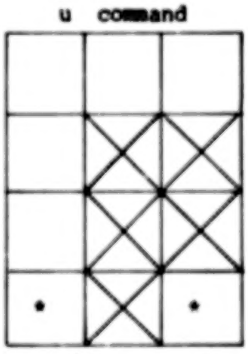
*Quantities have minor effect on element.



*θ undershoot, 12 percent.



*γ undershoot, 28 percent.



*20 percent u overshoot for $b_{4,1}$; 20 percent u undershoot for $b_{4,3}$.

Key:

- Value cannot change
- No significant effect on decoupling
- θ γ u — Decoupling lost for θ, γ, and u, respectively
- θ γ u — Partial decoupling lost for θ, γ, and u, respectively
- * — The terms "overshoot" and "undershoot" refer to steady-state error.

TABLE VII.- SUMMARY OF EFFECTS OF INDIVIDUAL MEASUREMENT ERRORS ON DECOUPLING PROCESS

[Blank blocks indicate no significant effect]

Measurement error in -	Response commanded	Effect on -		
		θ	γ	u
θ	θ	Large	Small	Small
	γ			
	u			
$\dot{\theta}$	θ			
	γ			
	u			
α	θ	Negligible	Small	Small
	γ	Small	Large	Small
	u			
u	θ			
	γ			
	u	Negligible	Small	Large

TABLE VIII.- COMPARISON OF EFFECTS OF COMBINED MEASUREMENT ERRORS ON DECOUPLING PROCESS

[All control differences from nominal are small and less than those shown in time history in figure 8 for 40-percent error in θ]

(a) 3° command in θ

Condition	Maximum response in -		
	θ, deg	γ, deg	u, m/sec
Positive 20-percent error in θ, $\dot{\theta}$, α, and u	2.5	-0.15	0.035
Positive 20-percent error in θ, $\dot{\theta}$, and u; negative 20-percent error in α	2.4	-.90	.110
Positive 20-percent error in θ only	2.5	-.45	.075
Negative 20-percent error in θ only	3.7	.70	-.120

(b) 6° command in γ

Condition	Maximum response in -		
	γ, deg	θ, deg	u, m/sec
Positive 20-percent error in θ, $\dot{\theta}$, α, and u	5.3	-0.11	0.070
Positive 20-percent error in θ, $\dot{\theta}$, and u; negative 20-percent error in α	6.7	.14	-.080
Positive 20-percent error in α only	5.3	-.13	.085
Negative 20-percent error in α only	6.7	.15	-.100

TABLE IX.- EFFECT OF SPEED ON A AND B MATRICES

[Multiplication factors for effects in A and B matrices due to speed changes. (Blank blocks denote no change)]

	A				B		
	θ'	q'	α'	u'	δ'_{th}	δ'_{t}	δ'_{f}
$d\theta'/dt$							
$d^2\theta'/dt^2$		u/u_0	$(u/u_0)^2$	u/u_0	$(u/u_0)^2$	$(u/u_0)^2$	$(u/u_0)^2$
$d\alpha'/dt$			u/u_0		u/u_0	u/u_0	u/u_0
du'/dt	u/u_0		u/u_0		u/u_0	u/u_0	u/u_0

TABLE X.- EFFECT OF CHANGE IN FLIGHT CONDITION ON DECOUPLING PROCESS

[Blank blocks denote no effect]

	Speed increased by 50 percent[a]	Speed increased by 100 percent	Speed decreased by 50 percent
Maximum response due to 1.5 m/sec command in u			
u, m/sec	1.54 (2.44)	1.58	1.35 (sluggish)
θ, deg	-0.055 (0.017)	-0.09	0.15
γ, deg	-0.63 (-0.20)	-1.0	1.4
Maximum response due to 3° command in θ			
u, m/sec		0.05	
θ, deg	3.0 (3.0)	3.0	3.5
γ, deg		-0.025	
Maximum response due to 6° command in γ			
u, m/sec			
θ, deg			
γ, deg	6.0 (6.0)	6.0	6.0 (sluggish)

[a]Values in parentheses from $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$ case.

TABLE XI.- A AND B MATRICES WITH EFFECT OF LAG INCLUDED IN SYSTEM

(a) Lag in thrust[a]

$$A = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_{th} \end{array} \begin{array}{c} \begin{array}{ccccc} \theta & q & \alpha & u & \delta_{th} \end{array} \\ \begin{bmatrix} 0 & 1.0 & 0 & 0 & 0 \\ 0 & -1.23 & -0.52 & 0.225 & \begin{array}{c} 0 \\ (-0.146) \end{array} \\ 0 & 1.0 & -0.368 & -0.64 & \begin{array}{c} 0 \\ (-0.185) \end{array} \\ -0.3195 & 0 & 0.157 & -0.1018 & 0.1047 \\ 0 & 0 & 0 & 0 & -0.5 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_{th} \end{array} \begin{array}{c} \begin{array}{ccc} \delta_{th,1} & \delta_t & \delta_f \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2.38 & 0.1487 \\ 0 & -0.0676 & -0.1712 \\ 0 & -0.01406 & -0.1190 \\ 0.5 & 0 & 0 \end{bmatrix} \end{array}$$

[a]Values in parentheses from $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$ case.

TABLE XI.- Concluded

(b) Lag in horizontal-tail servo

$$A = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_t \\ \ddot{\delta}_t \end{array} \begin{array}{c} \begin{array}{cccccc} \theta & q & \alpha & u & \delta_t & \dot{\delta}_t \end{array} \\ \begin{bmatrix} 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & -1.23 & -0.52 & 0.225 & -2.38 & 0 \\ 0 & 1.0 & -0.368 & -0.64 & -0.0676 & 0 \\ -0.3195 & 0 & 0.157 & -0.1018 & -0.01406 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1046 & -95.4 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_t \\ \ddot{\delta}_t \end{array} \begin{array}{c} \begin{array}{ccc} \delta_{th} & \delta_{t,1} & \delta_f \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.1487 \\ 0 & 0 & -0.1712 \\ 0.1047 & 0 & -0.119 \\ 0 & 0 & 0 \\ 0 & 1046 & 0 \end{bmatrix} \end{array}$$

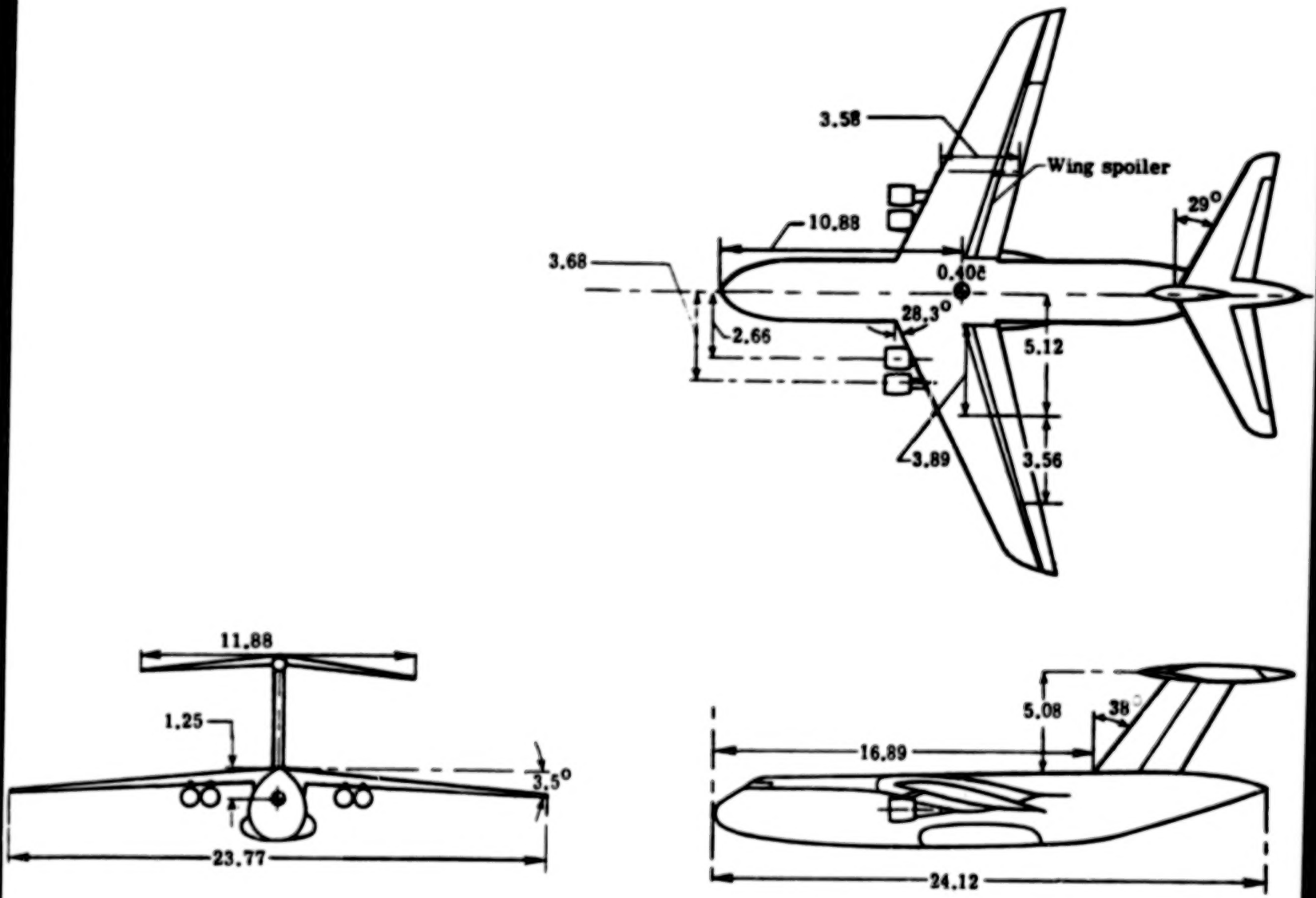
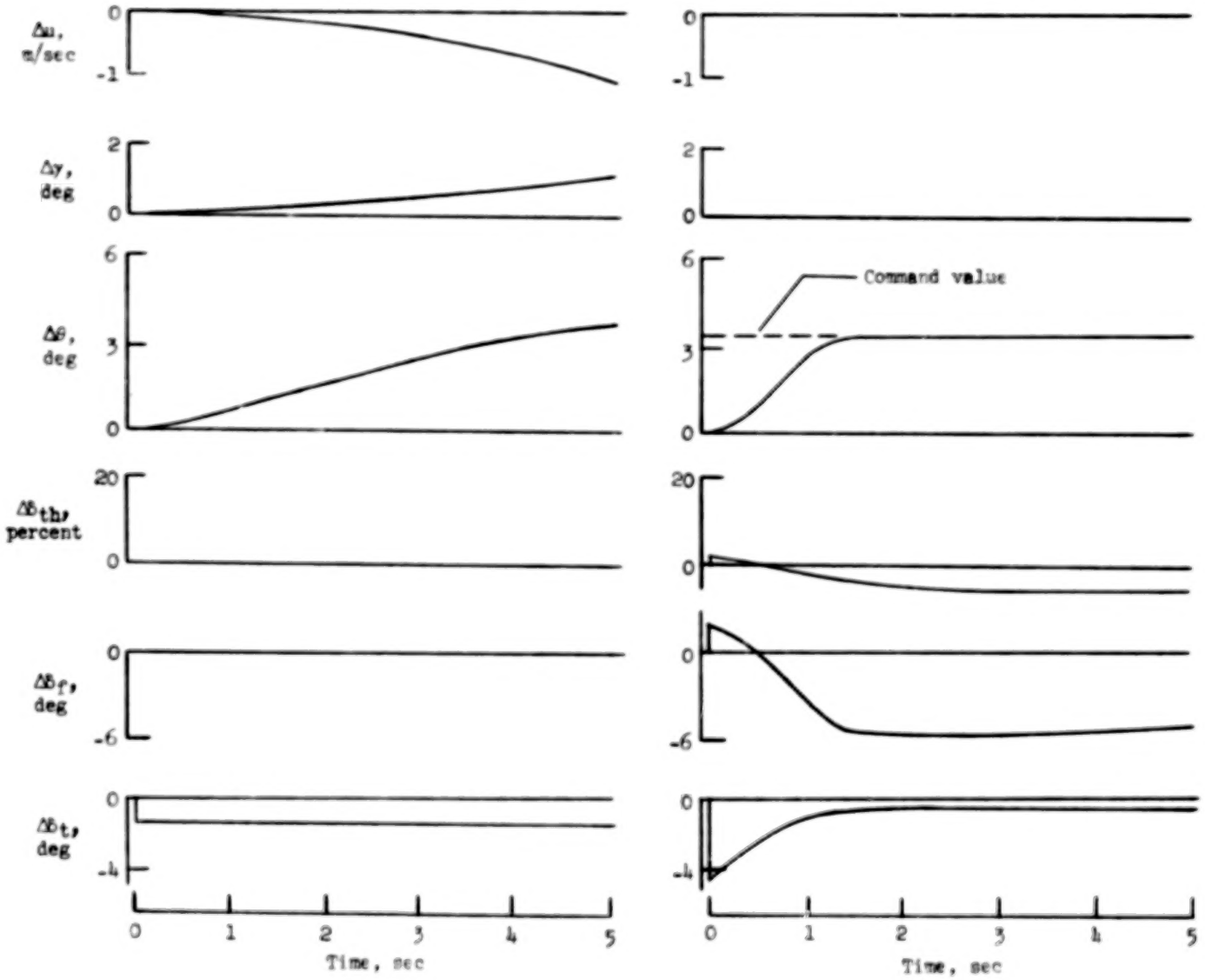


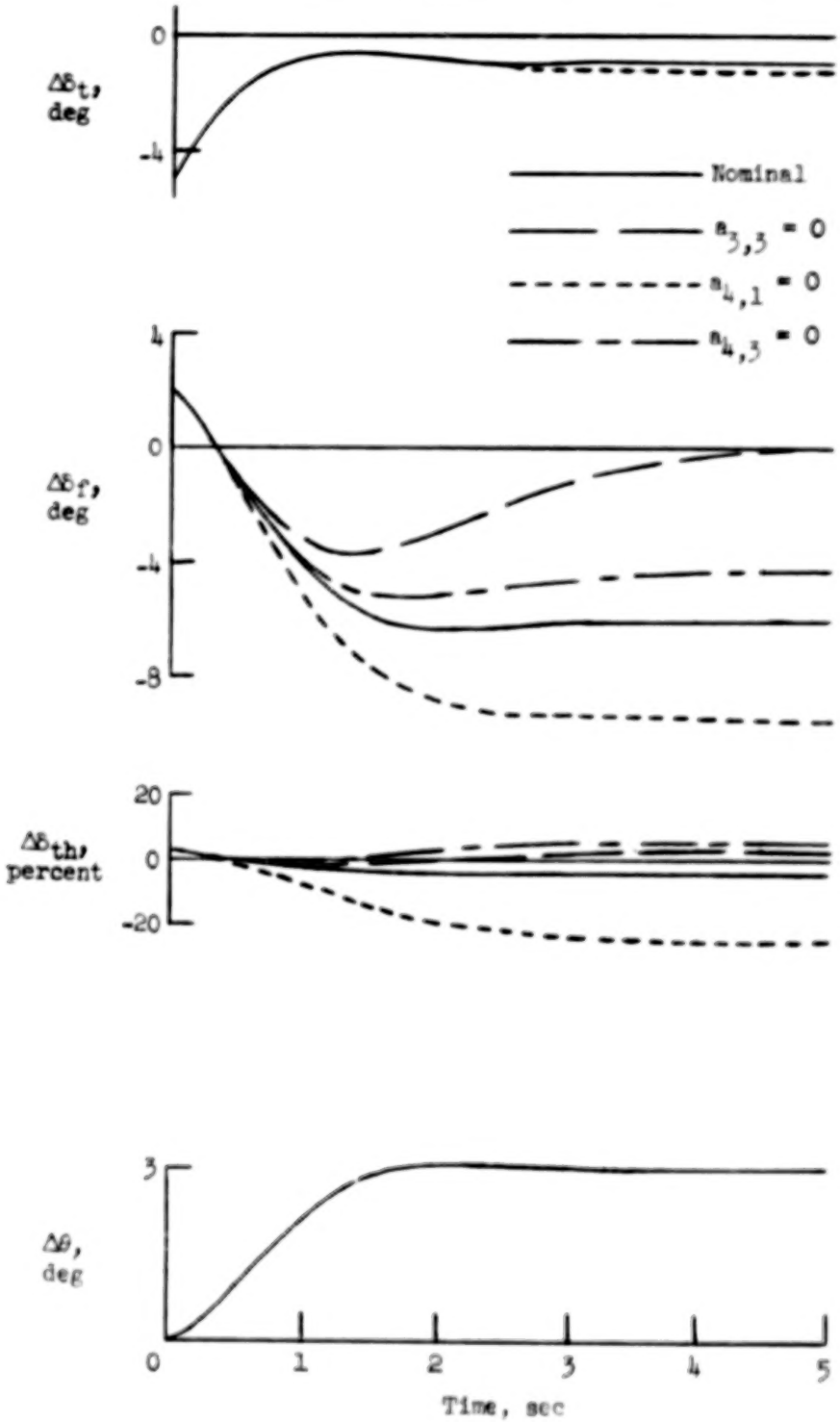
Figure 1.- Three-view drawing of simulated airplane.
(All linear dimensions are in m.)



(a) Basic response.

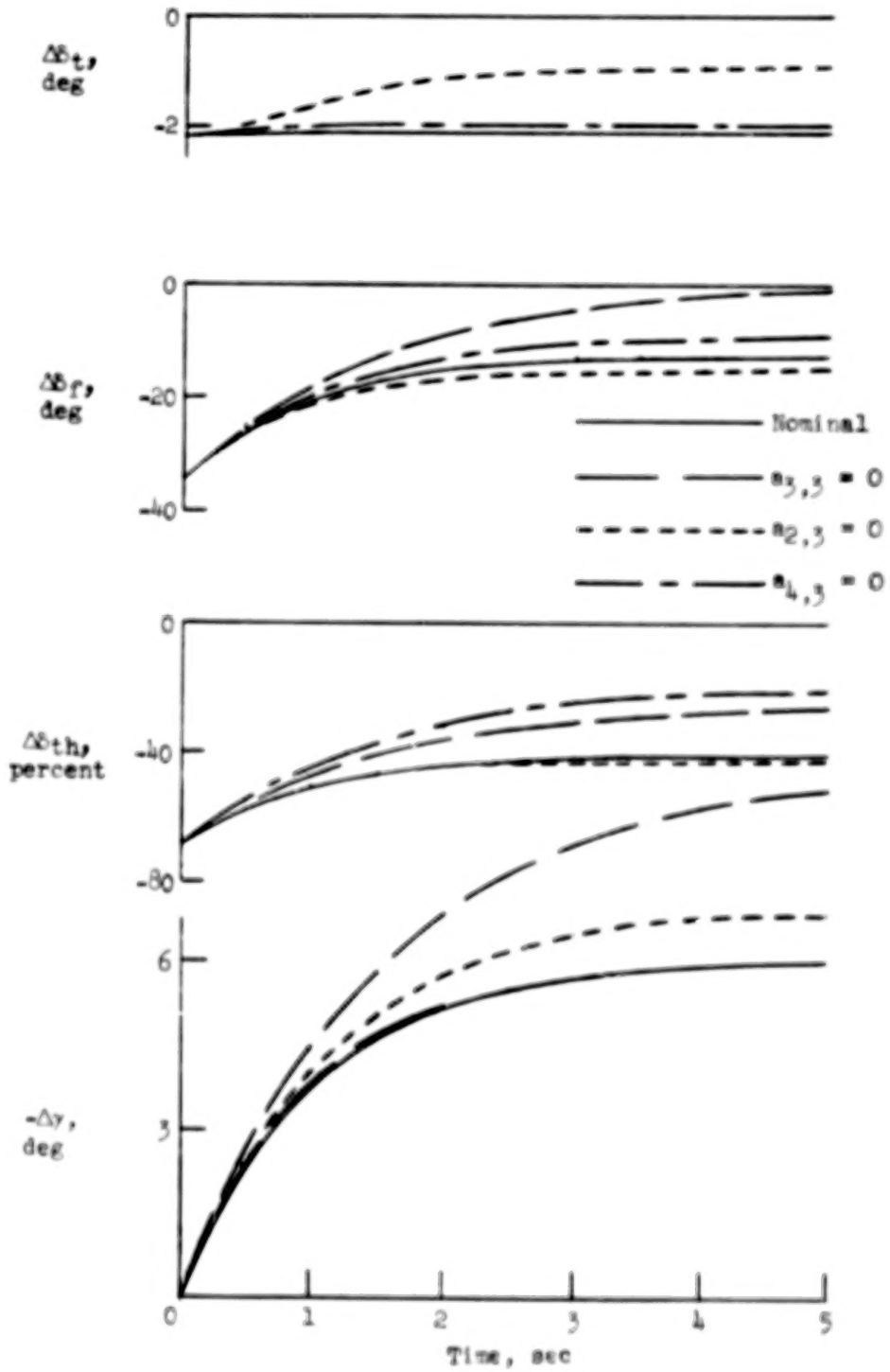
(b) Decoupled response.

Figure 2.- Comparison of pitch maneuvers performed by unaugmented and decoupled control systems. (Nominal $\alpha = 10^{o}$.)



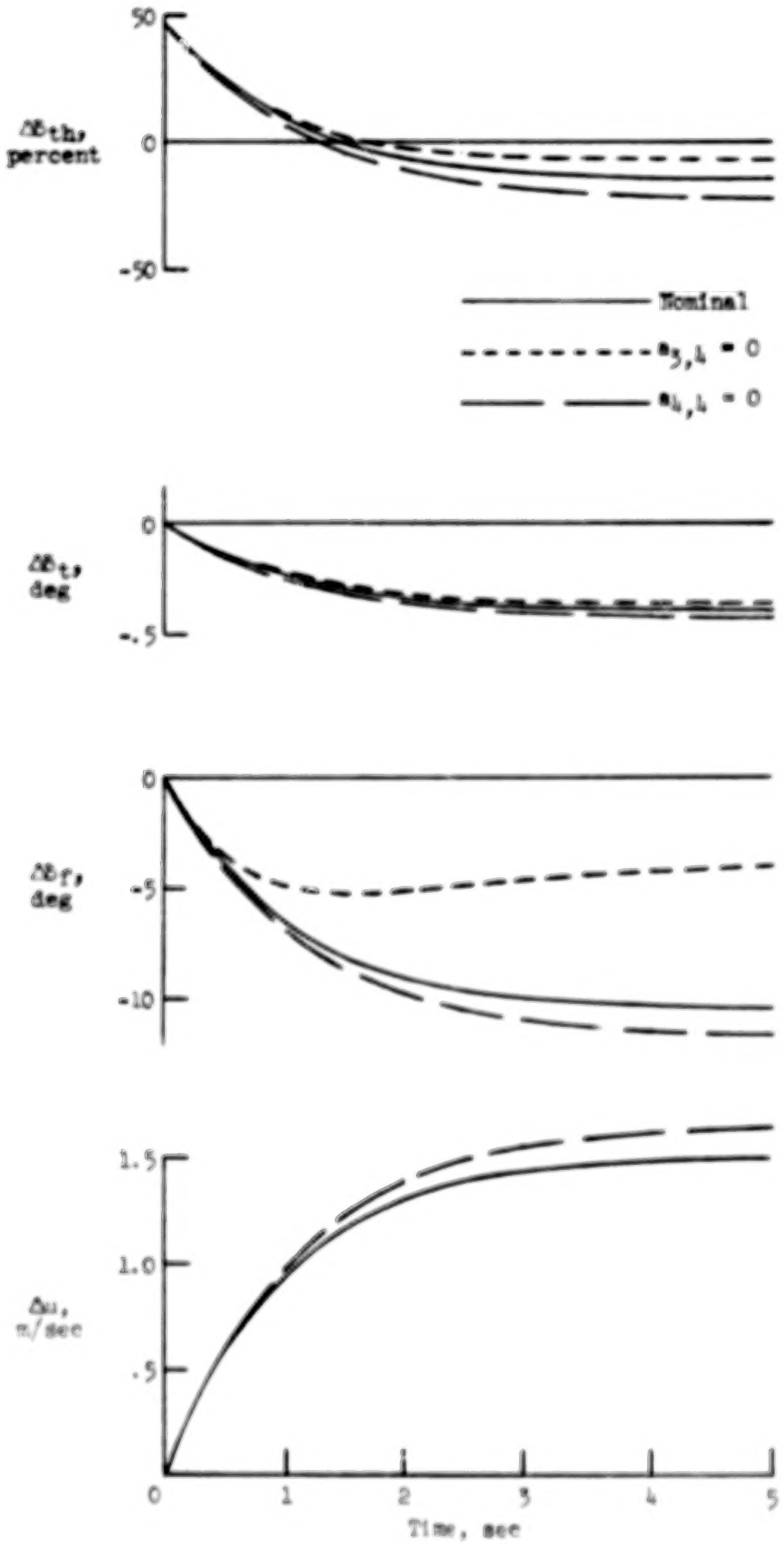
(a) 3^o pitch command.

Figure 3.- Effects of errors in A-matrix coefficients on control motions and commanded responses.



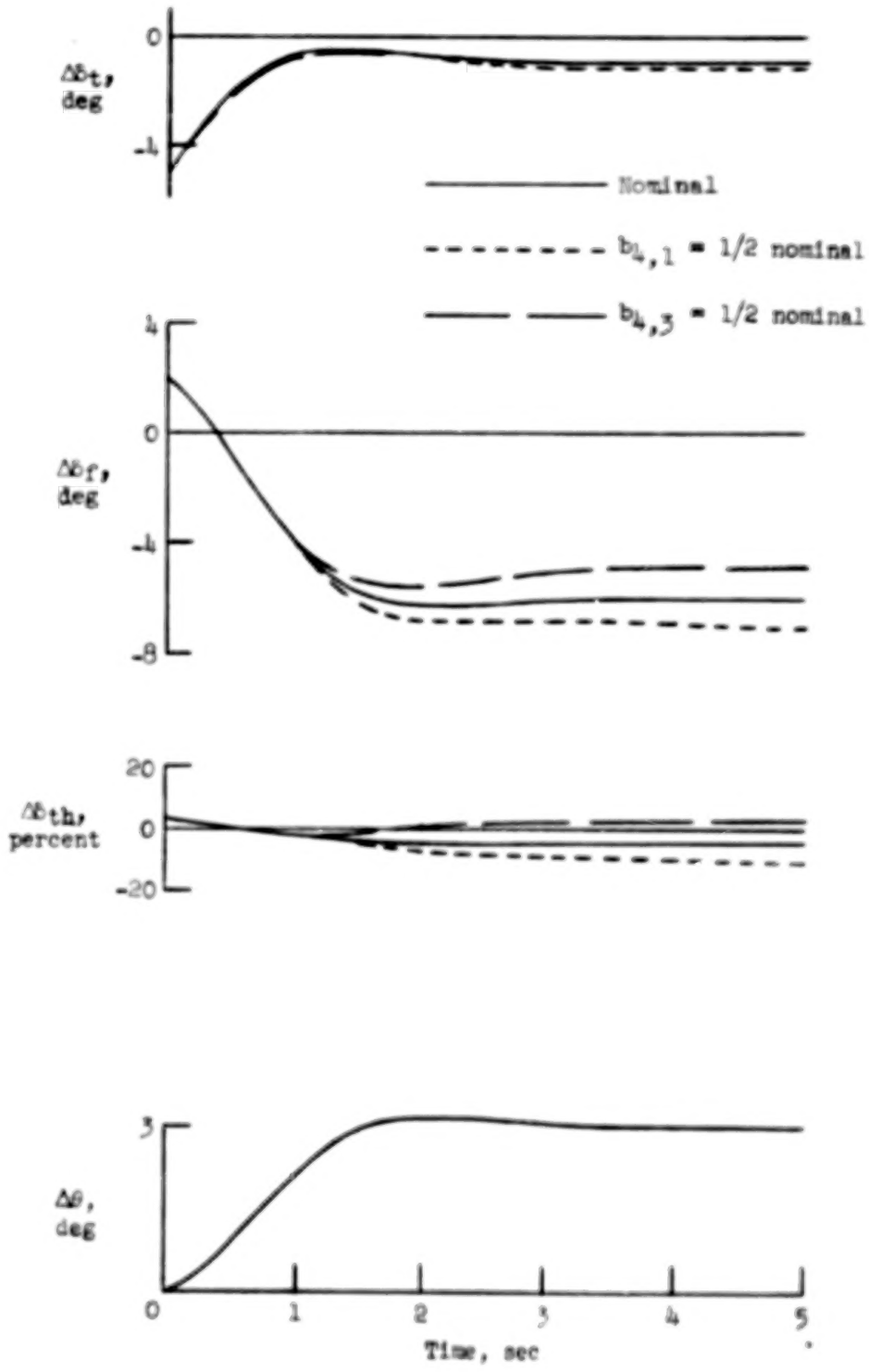
(b) 6° γ command.

Figure 3.- Continued.



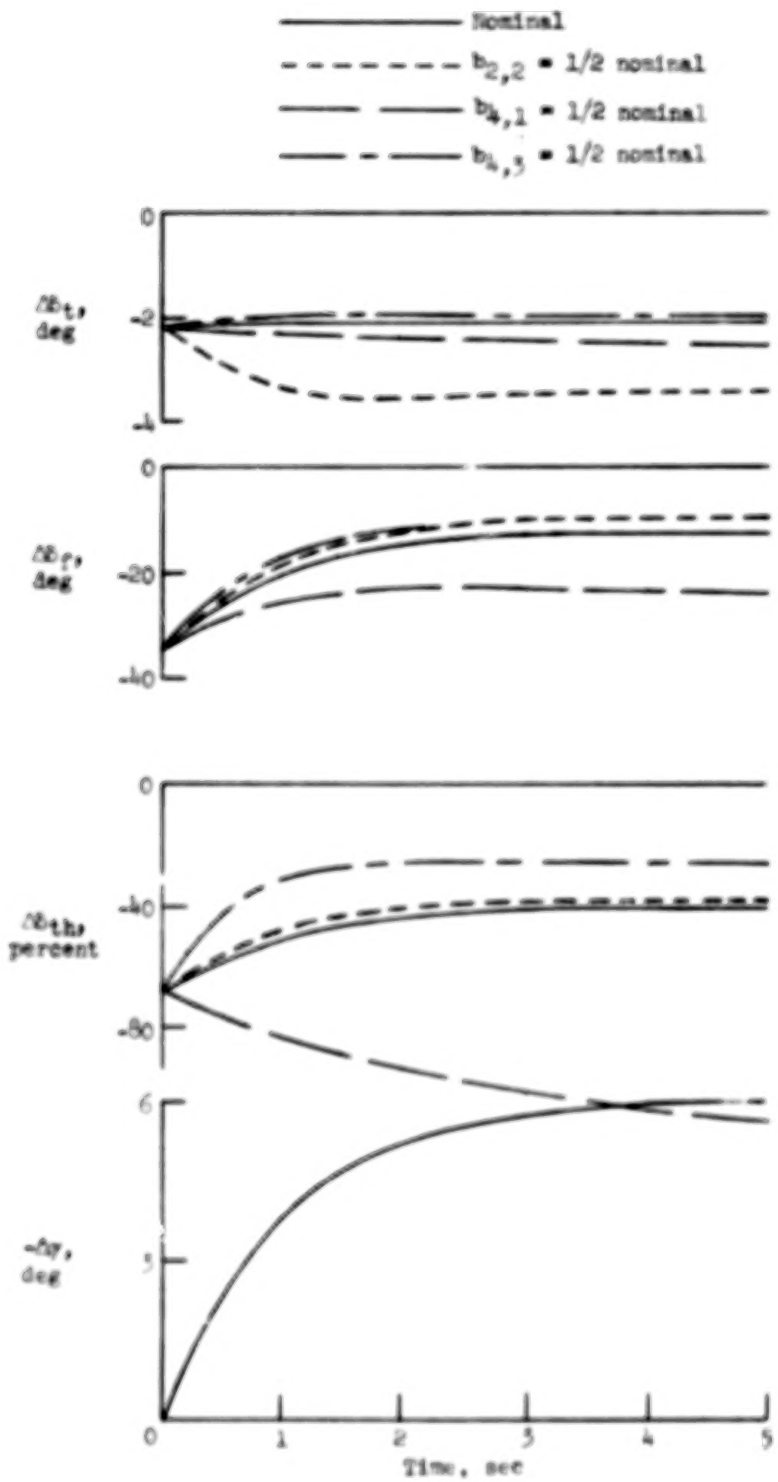
(c) Speed command of 1.5 m/sec.

Figure 3.- Concluded.



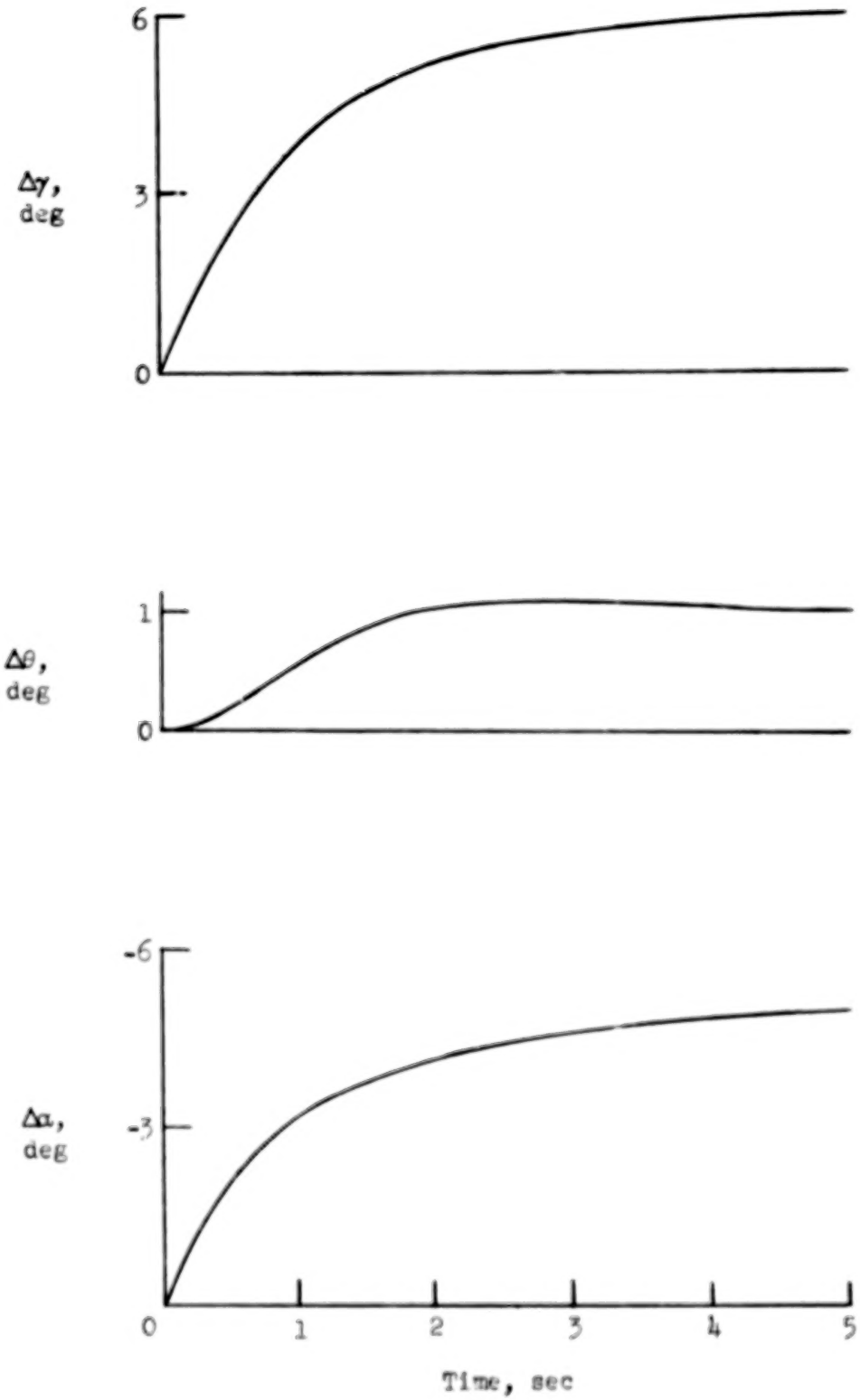
(a) 3° pitch command.

Figure 4.- Effects of errors in B-matrix coefficients on control motions and commanded responses.



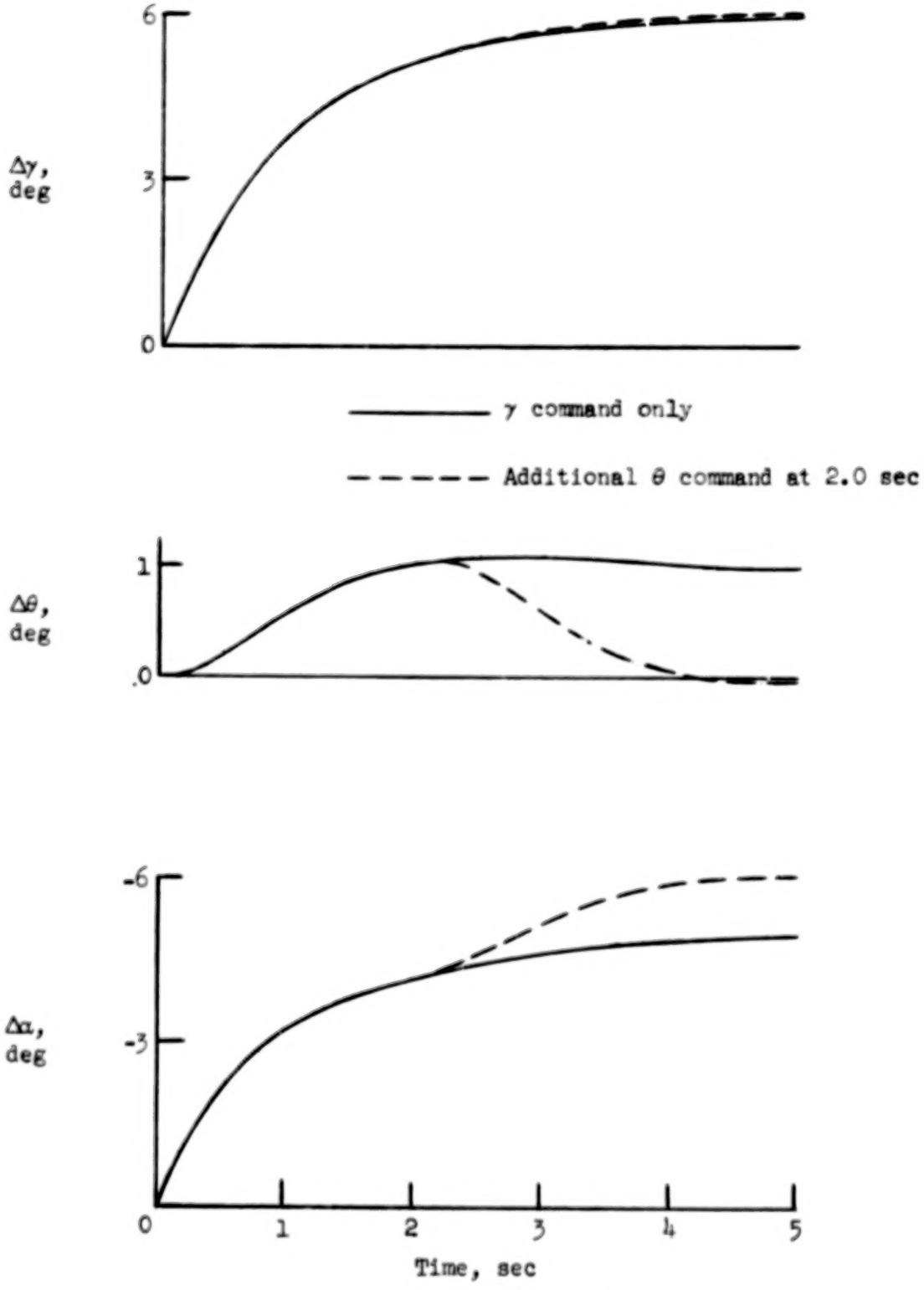
(b) 6° γ command.

Figure 4.- Concluded.



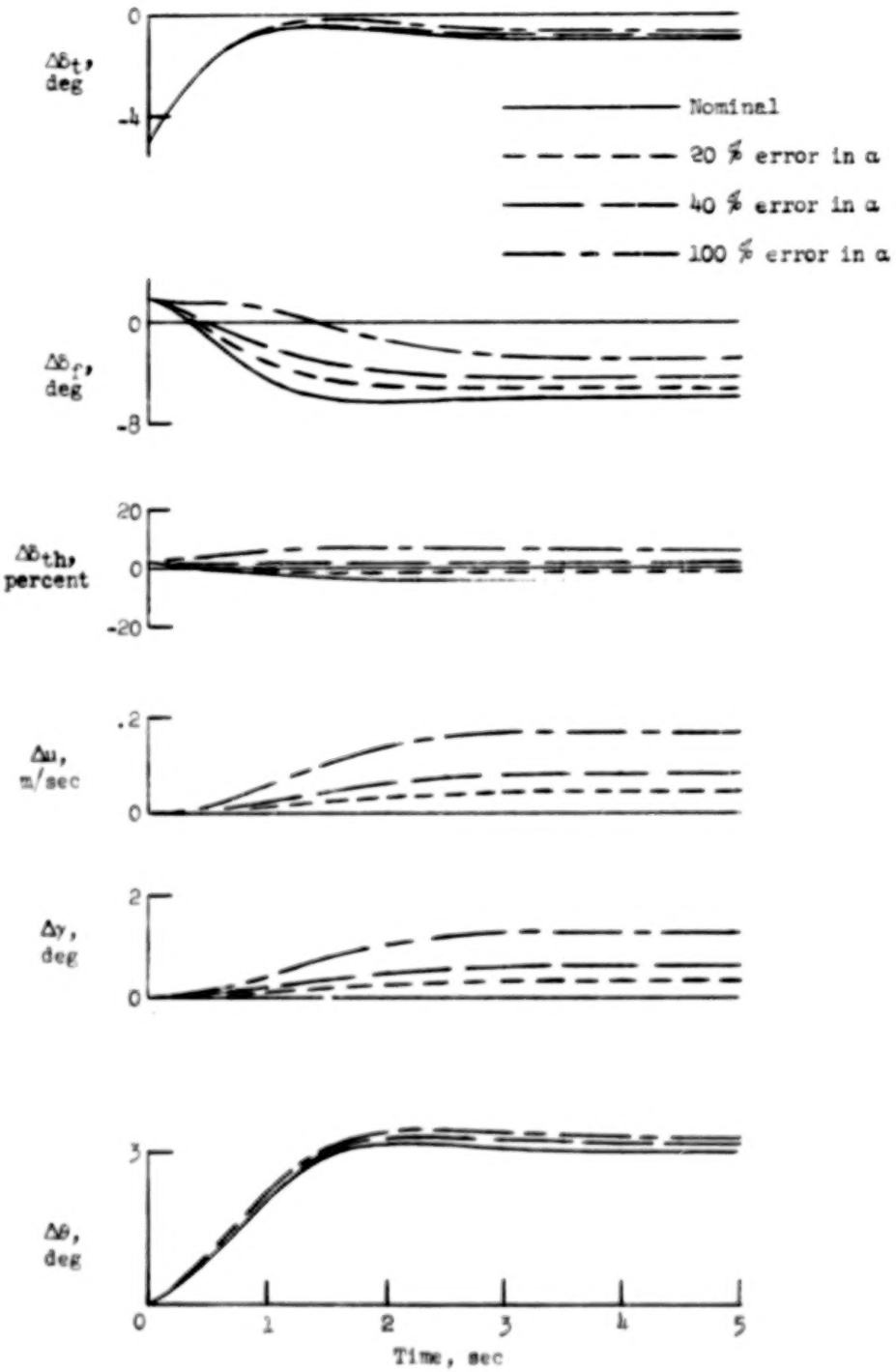
(a) No θ-command correction made by pilot.

Figure 5.- Effect of error in $b_{2,2}$ on θ and α for γ command of 6°; $b_{2,2}$ = 1/2 nominal.



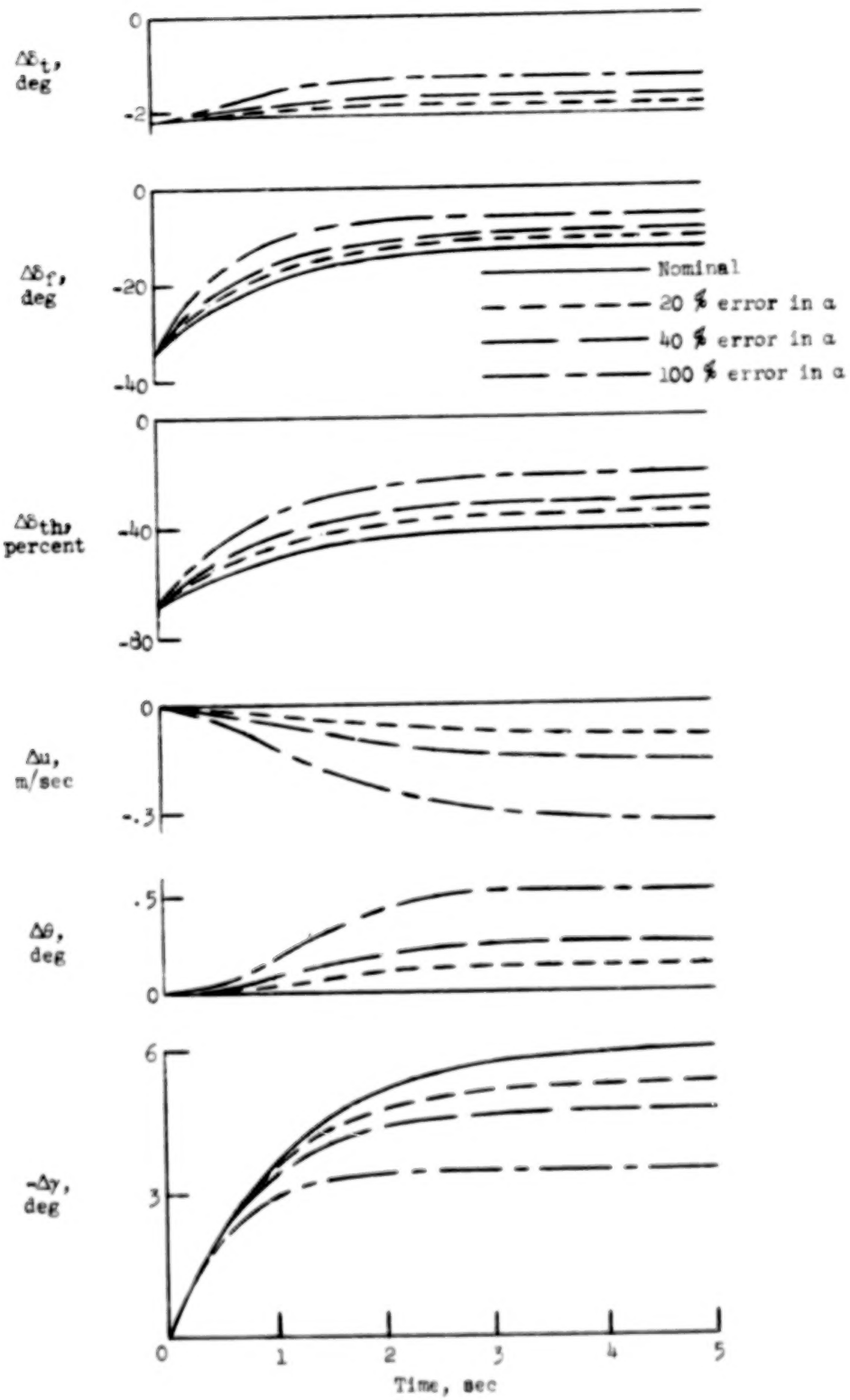
(b) θ-step command correction of 1.12° made by pilot.

Figure 5.- Concluded.



(a) 3° pitch command.

Figure 6.- Effect of α-feedback measurement error.



(b) 6° flight-path-angle command.

Figure 6.- Concluded.

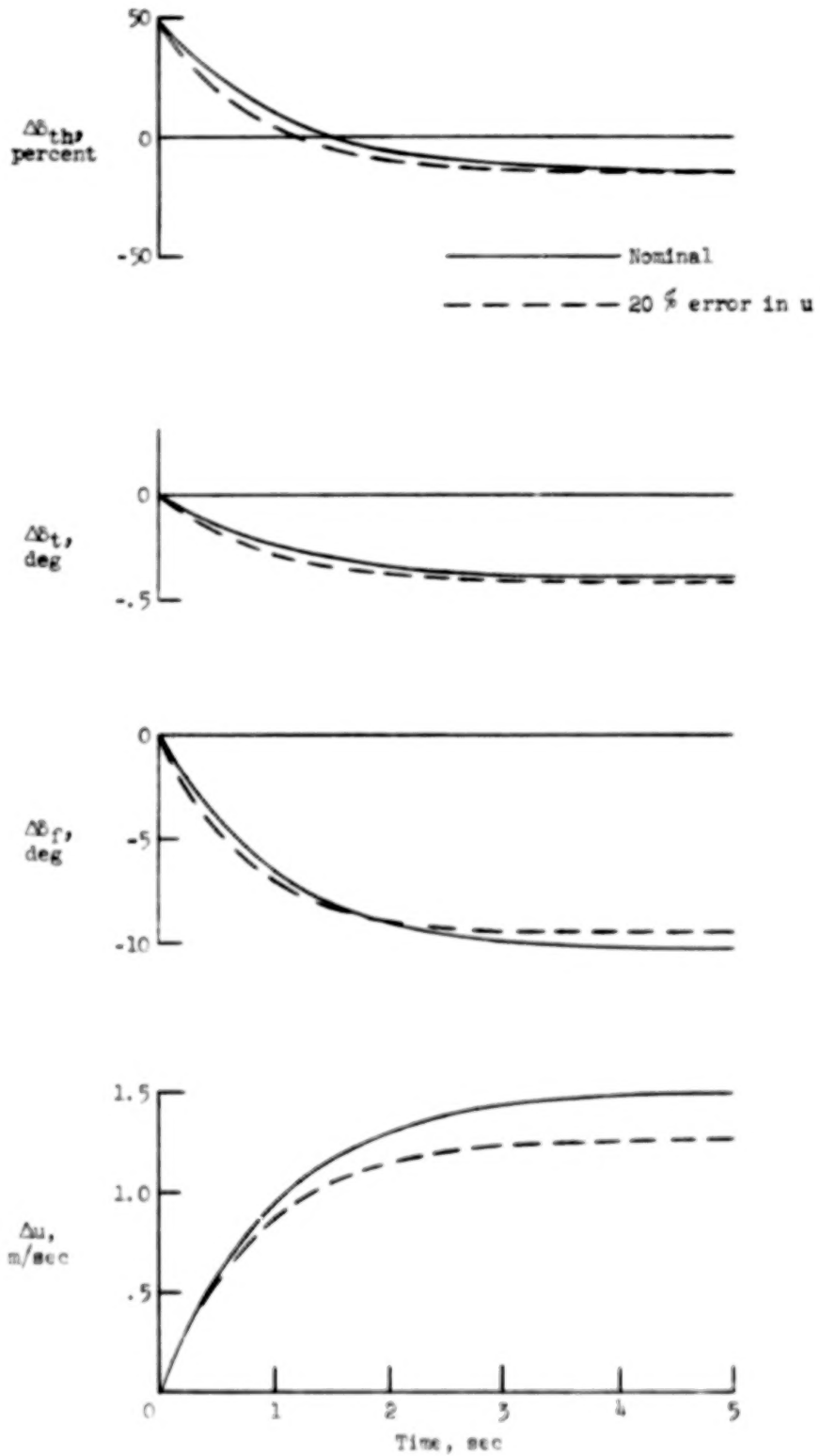


Figure 7.- Effect of u-feedback measurement error for 1.5 m/sec speed command.

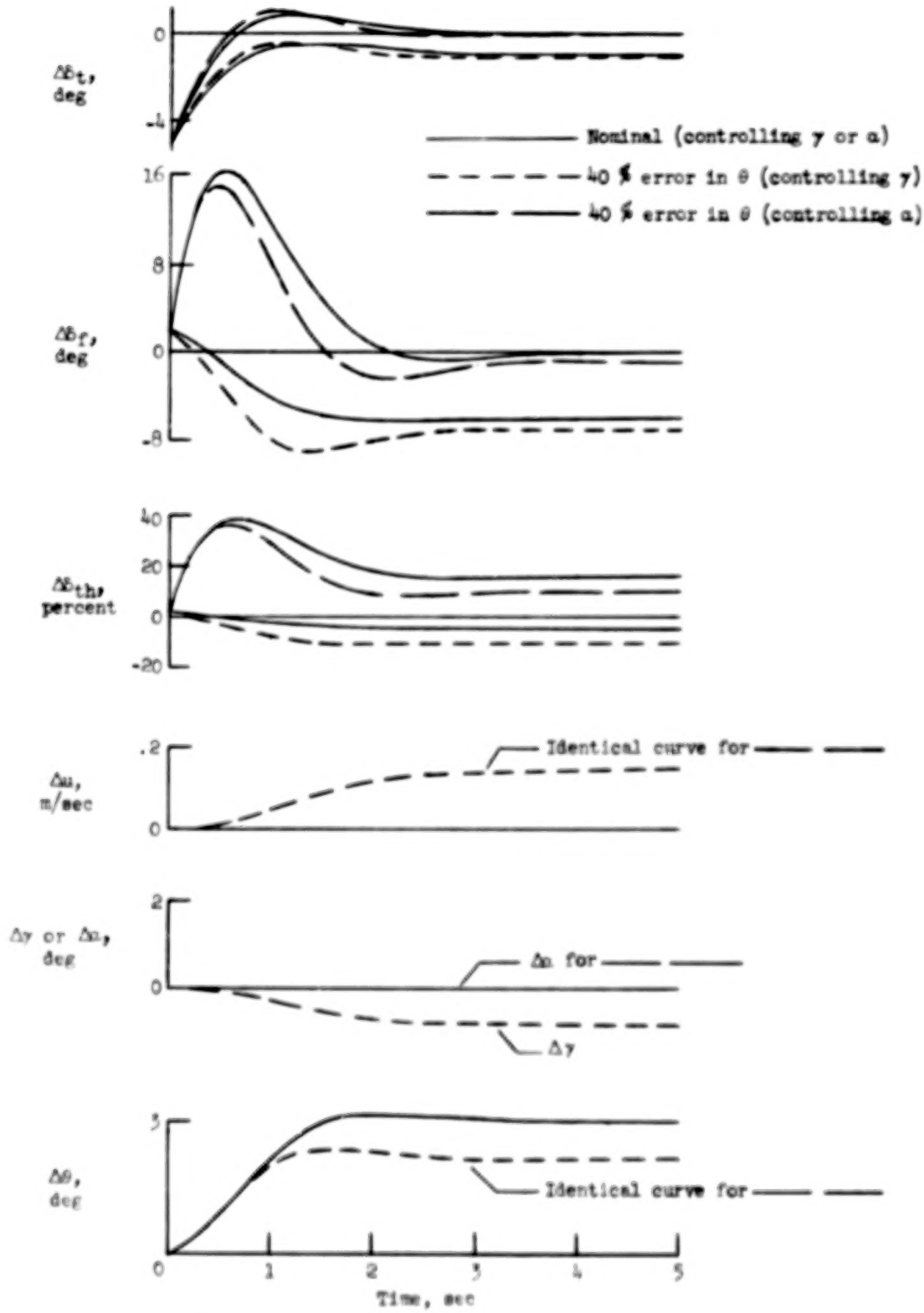


Figure 8.- Effect of θ-feedback measurement error for 3° pitch command for two decoupled designs.

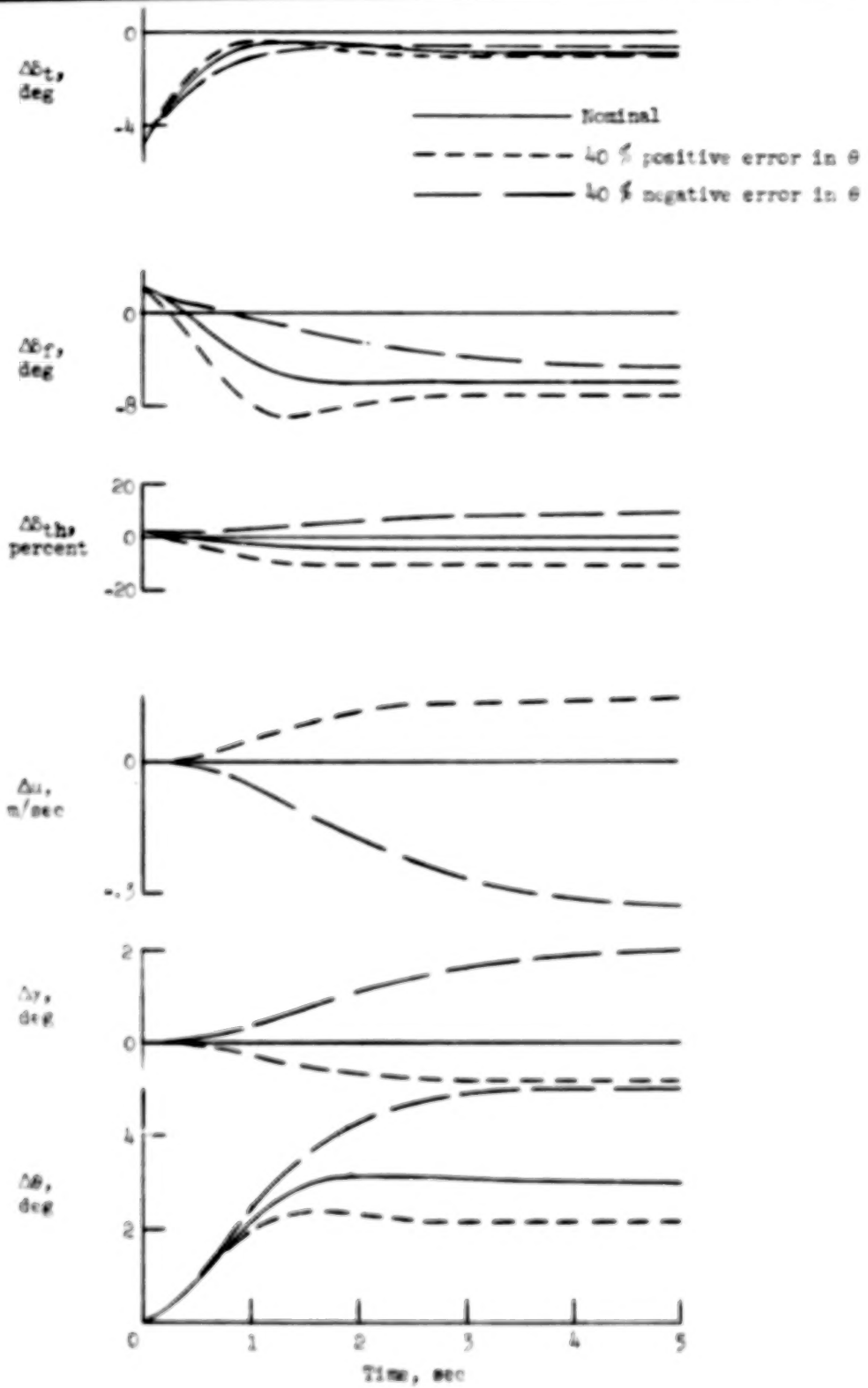


Figure 9.- Comparison of effects of positive and negative θ-feedback measurement error for 3° pitch command.

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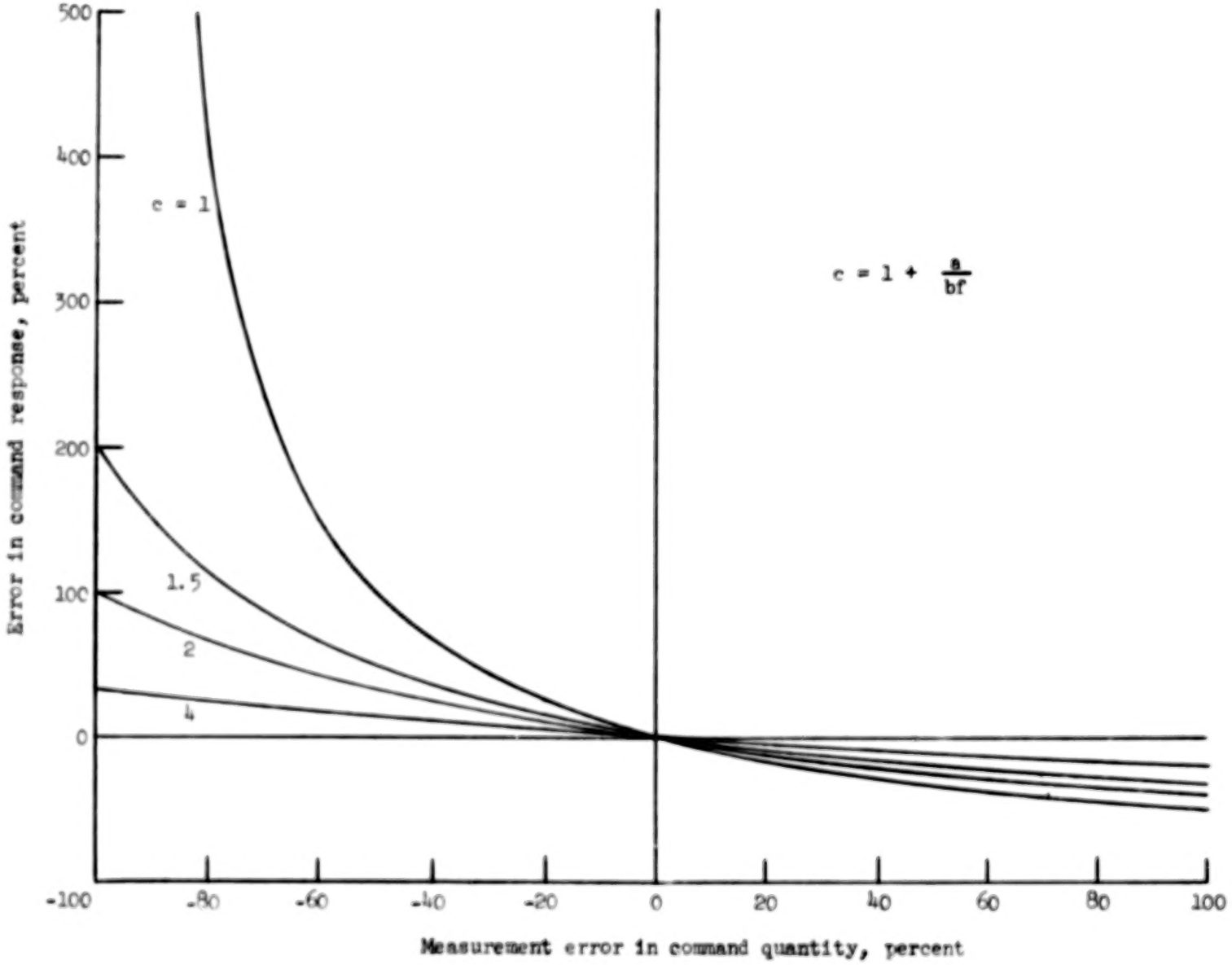


Figure 10.- Effect of measurement error in commanded response quantity.

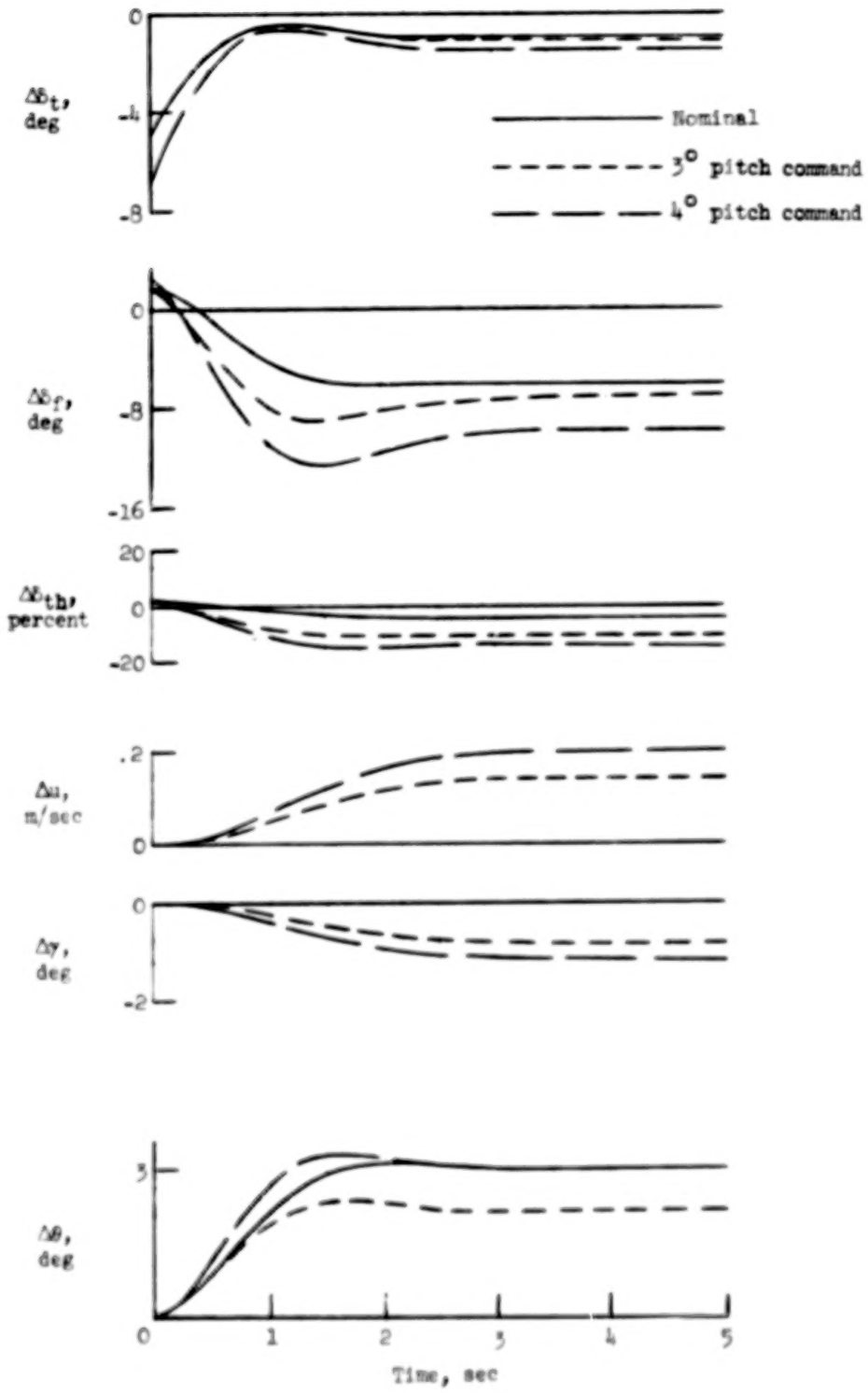


Figure 11.- Effect of compensating for measurement error. (Each command maneuver has 40-percent measurement error in θ.)

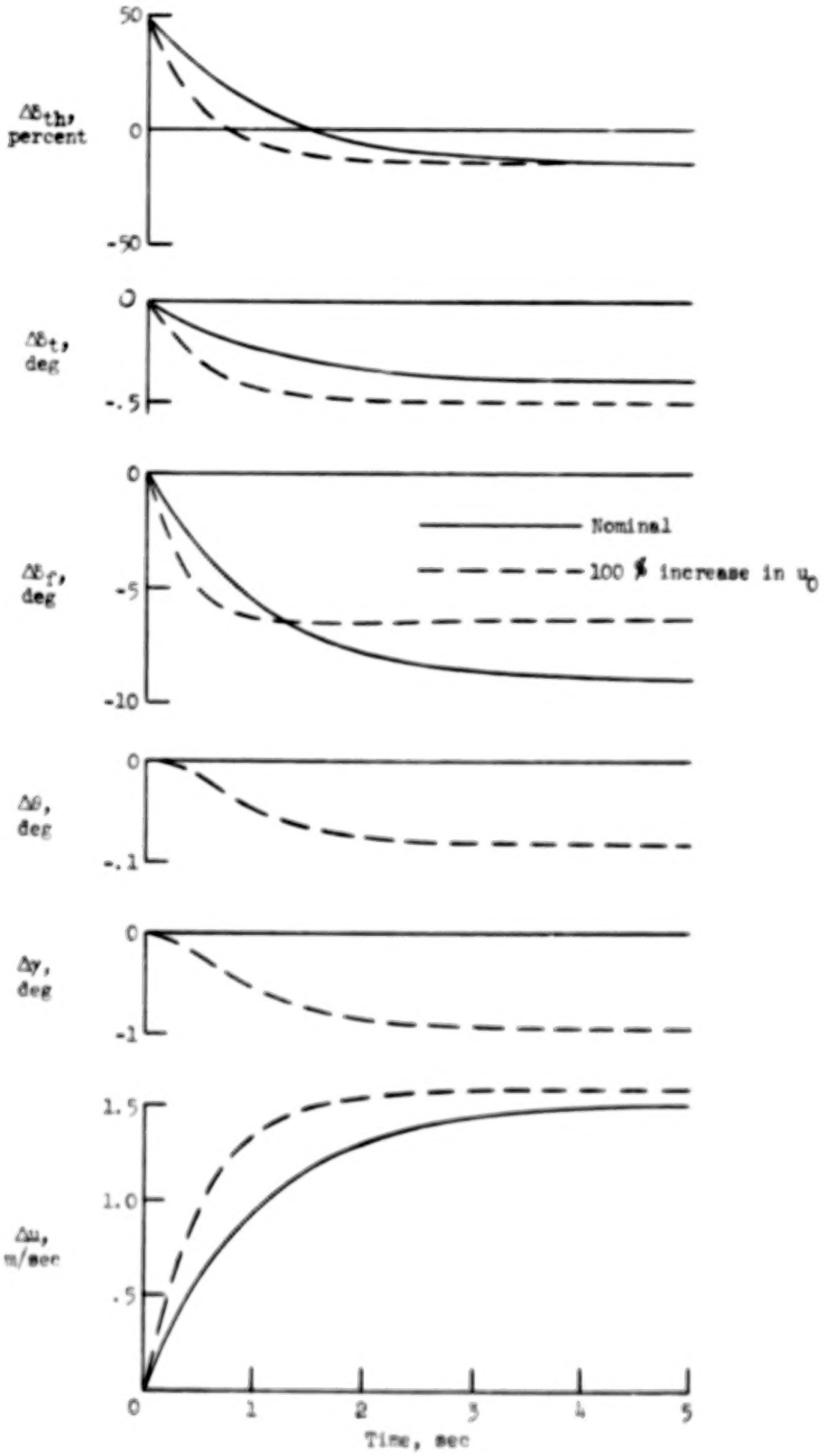
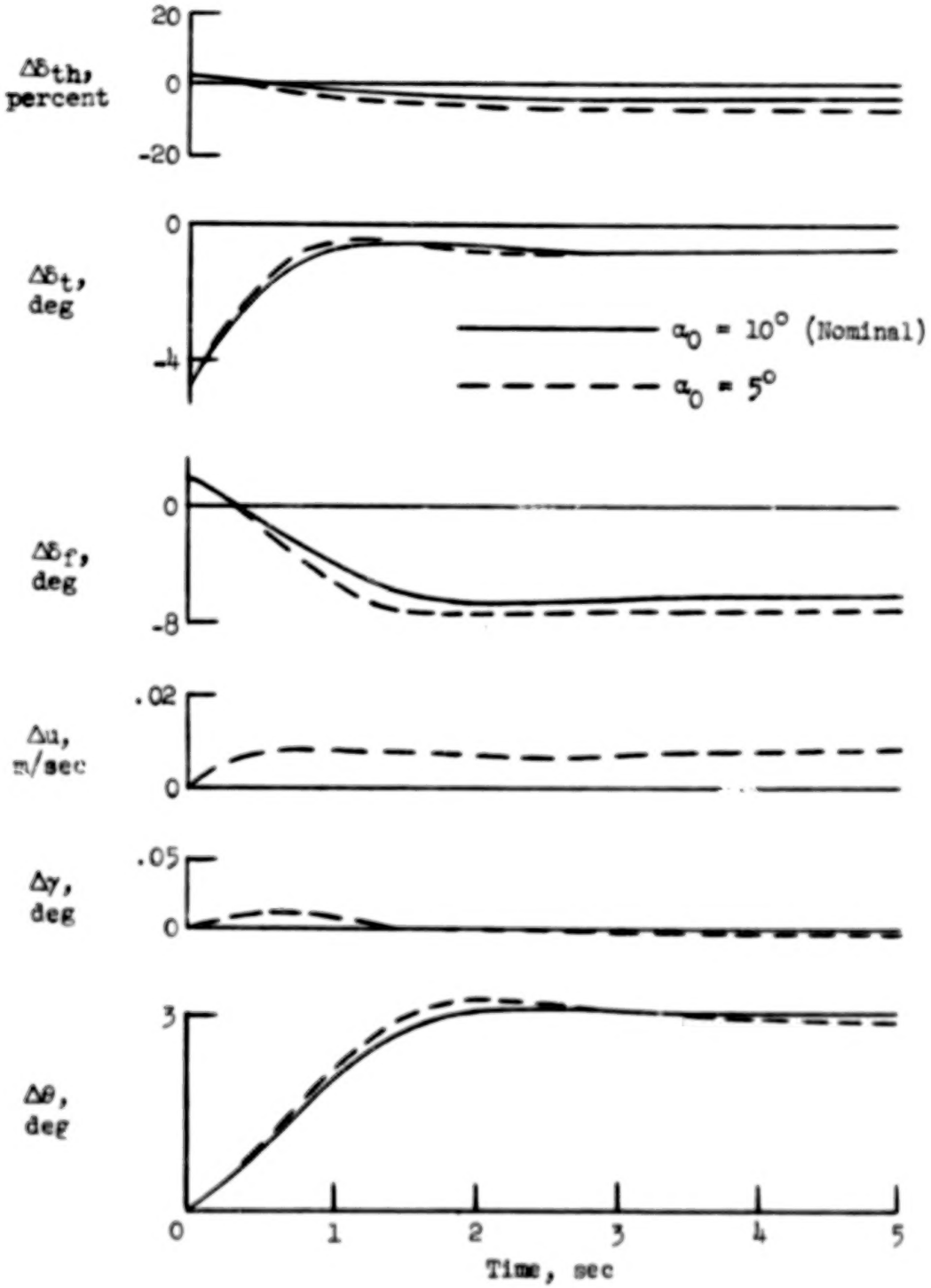
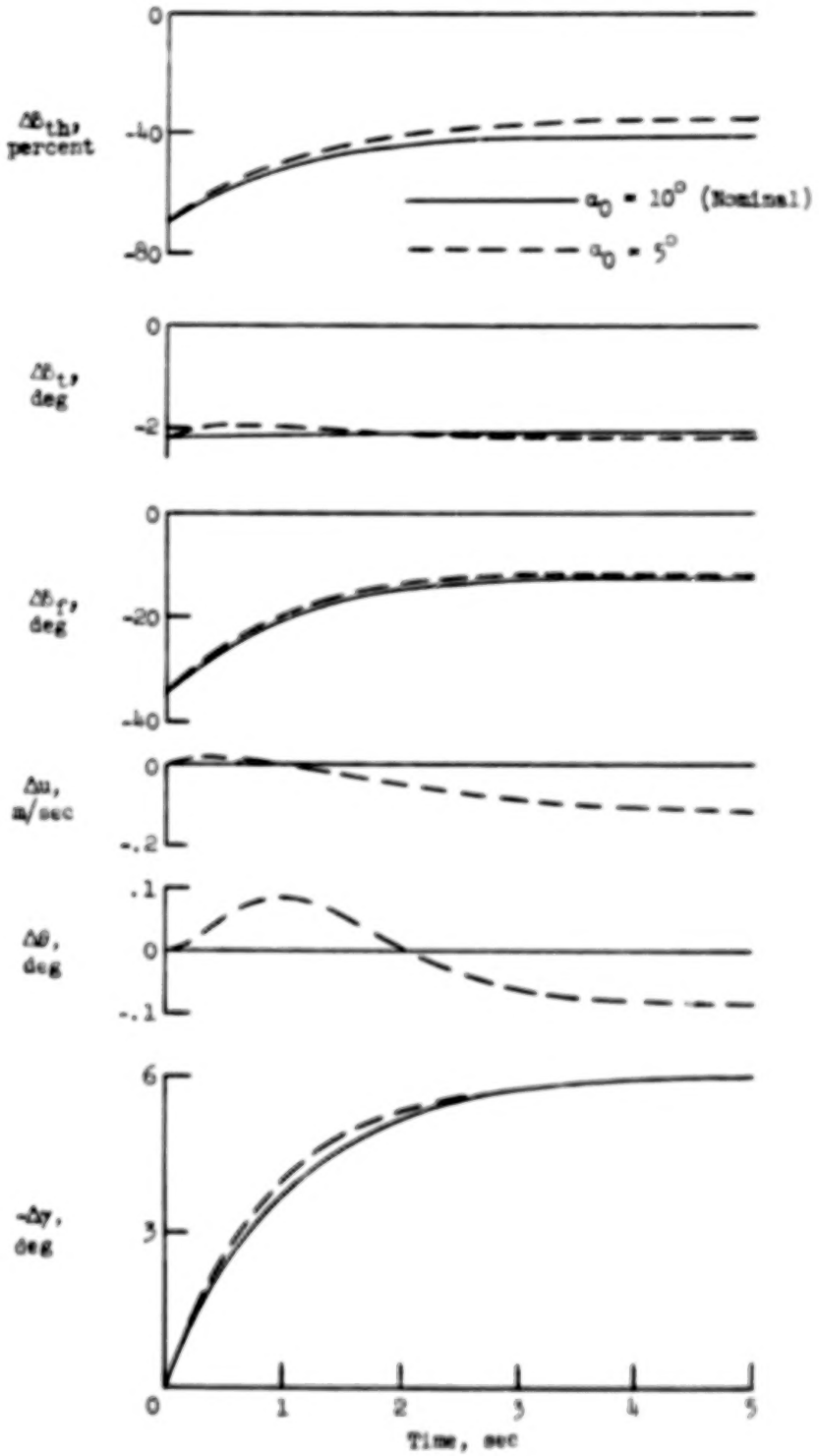


Figure 12.- Effect of change in flight condition for speed-command maneuver of 1.5 m/sec.



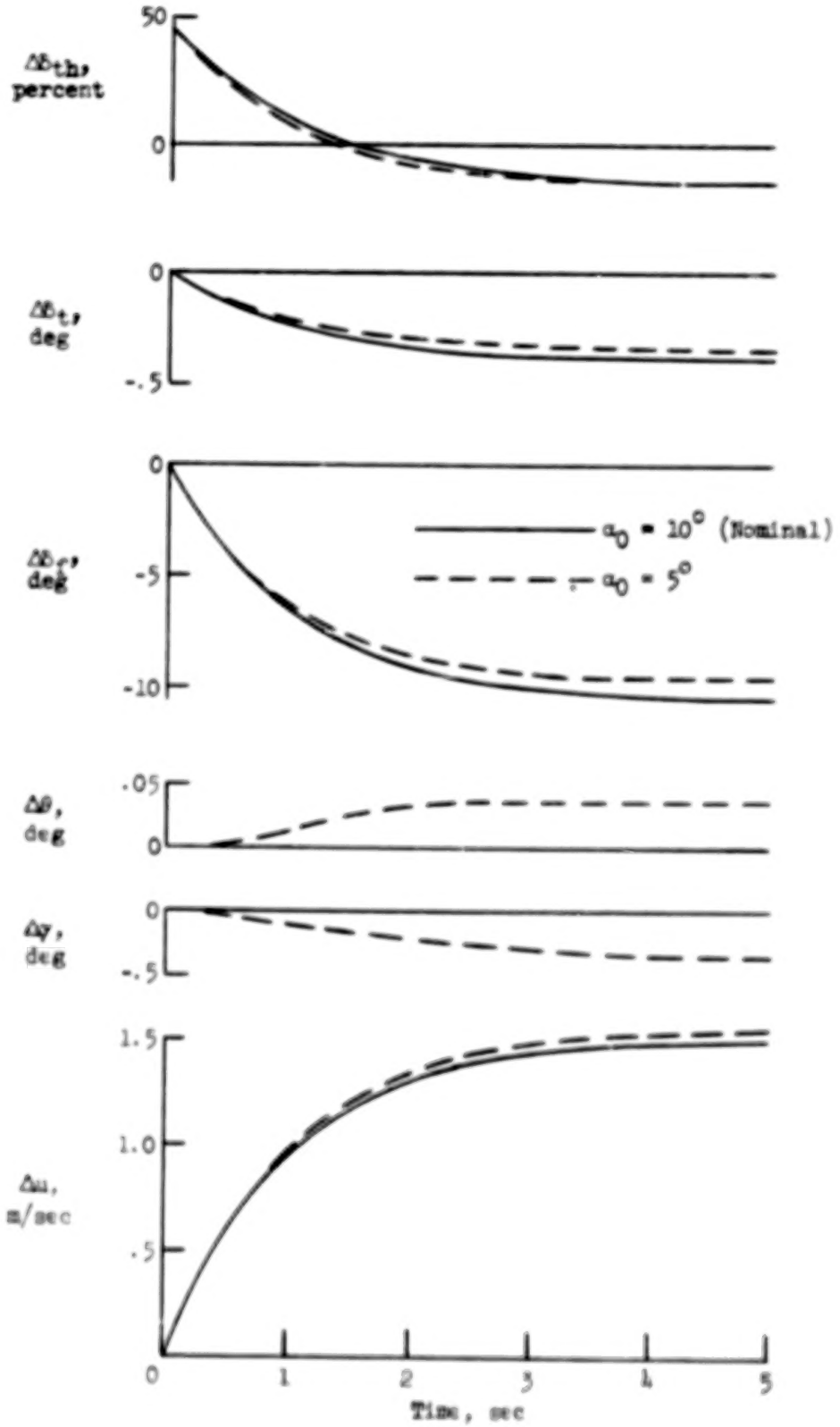
(a) 3° pitch command.

Figure 13.- Effect of change in flight condition on decoupling.



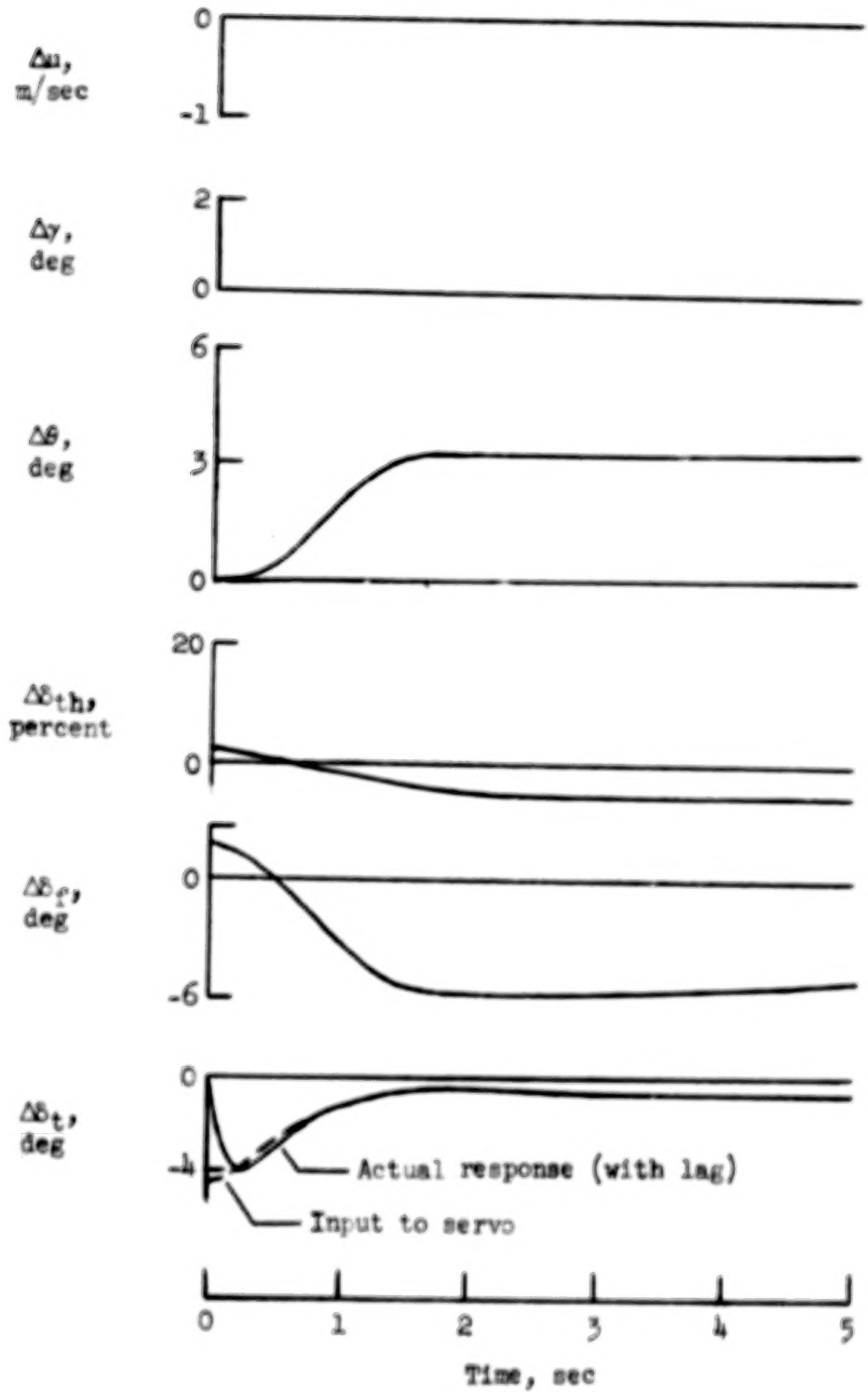
(b) 6° flight-path-angle command.

Figure 13.- Continued.



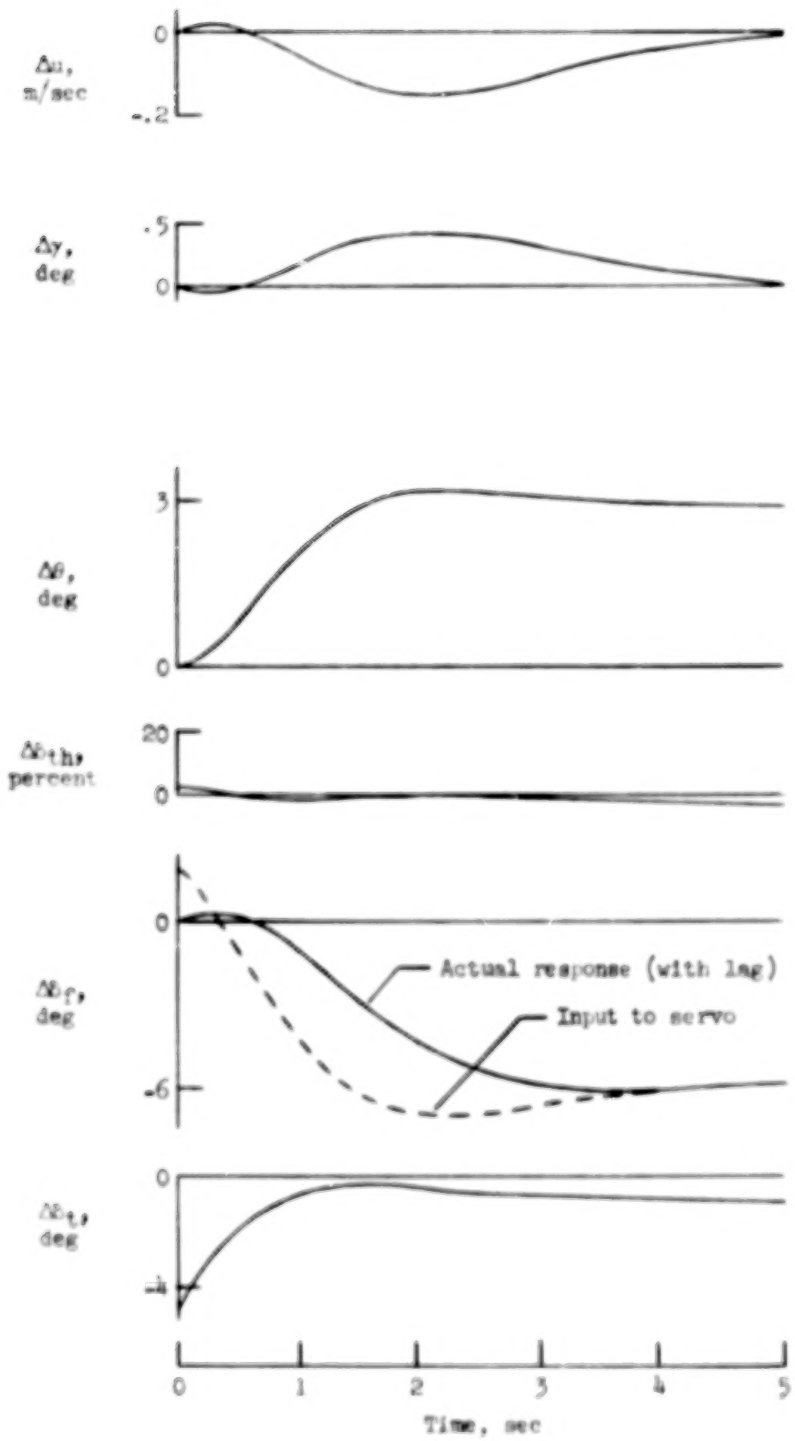
(c) Speed command of 1.5 m/sec.

Figure 13.- Concluded.



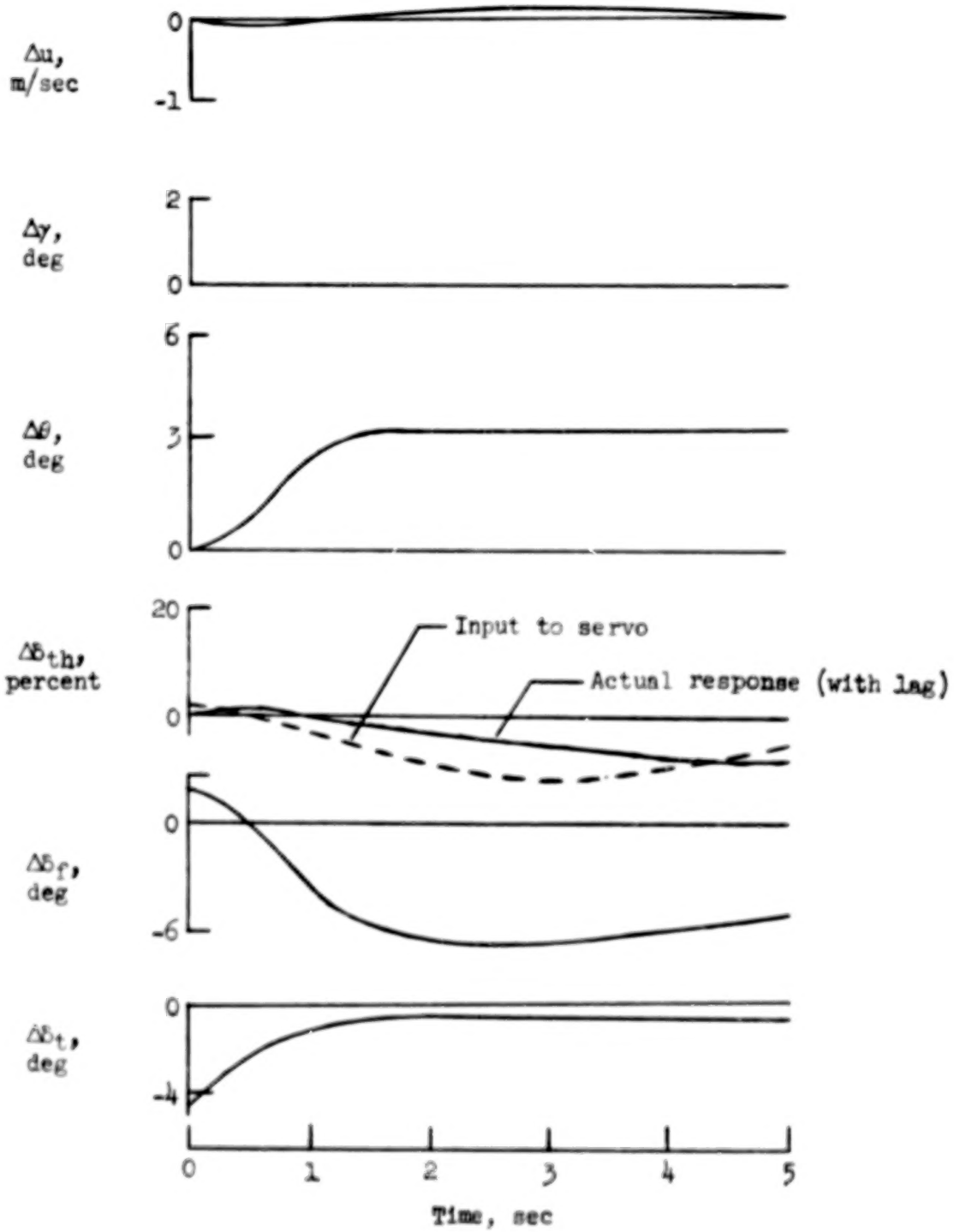
(a) Lag in horizontal-tail servo (ω_t = 32.3 rad/sec, ζ_t = 1.48).

Figure 14.- Effect of control lag for 3° pitch command.



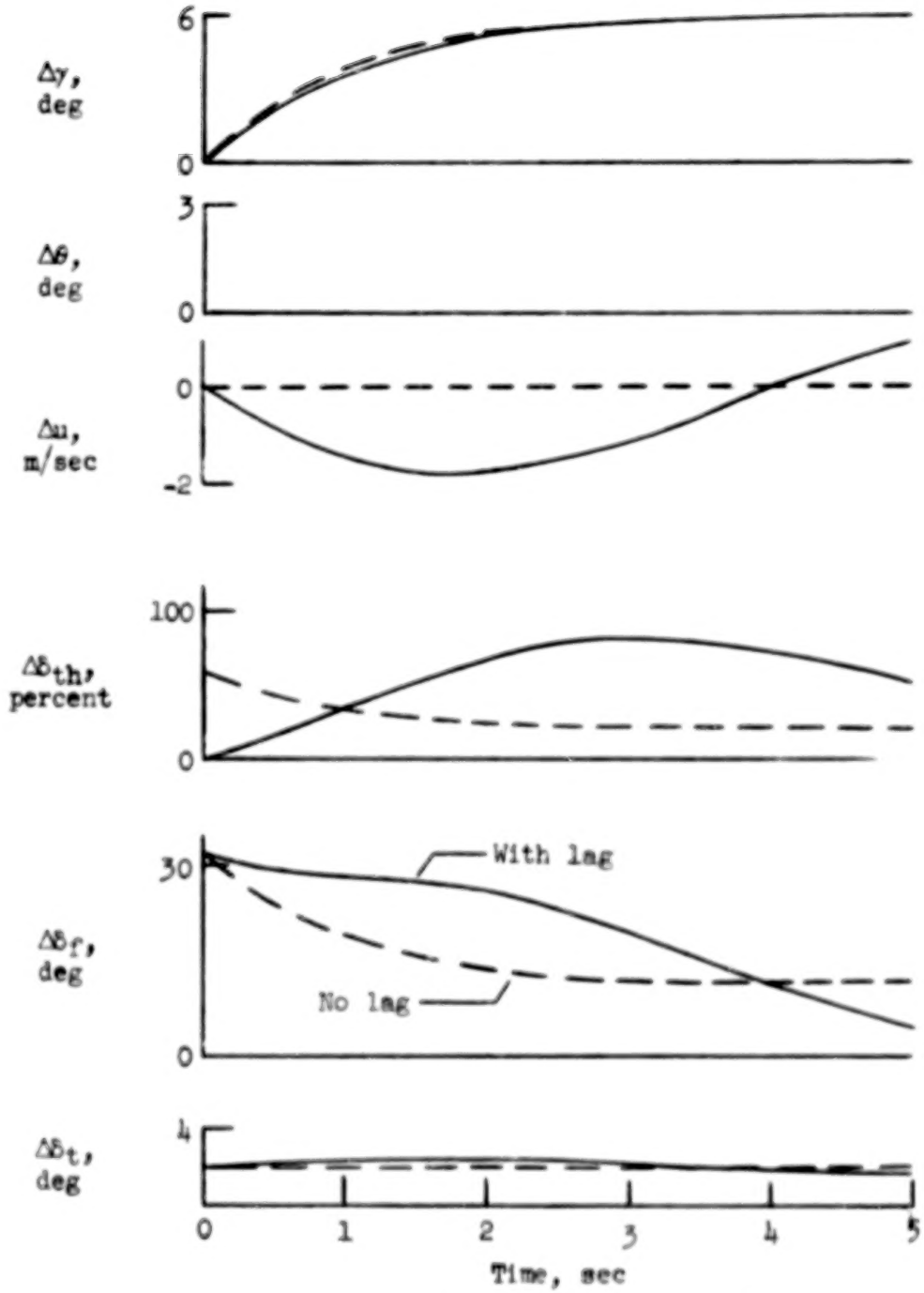
(b) Lag in flap servo (τ_f = 1.0 sec).

Figure 14.- Continued.



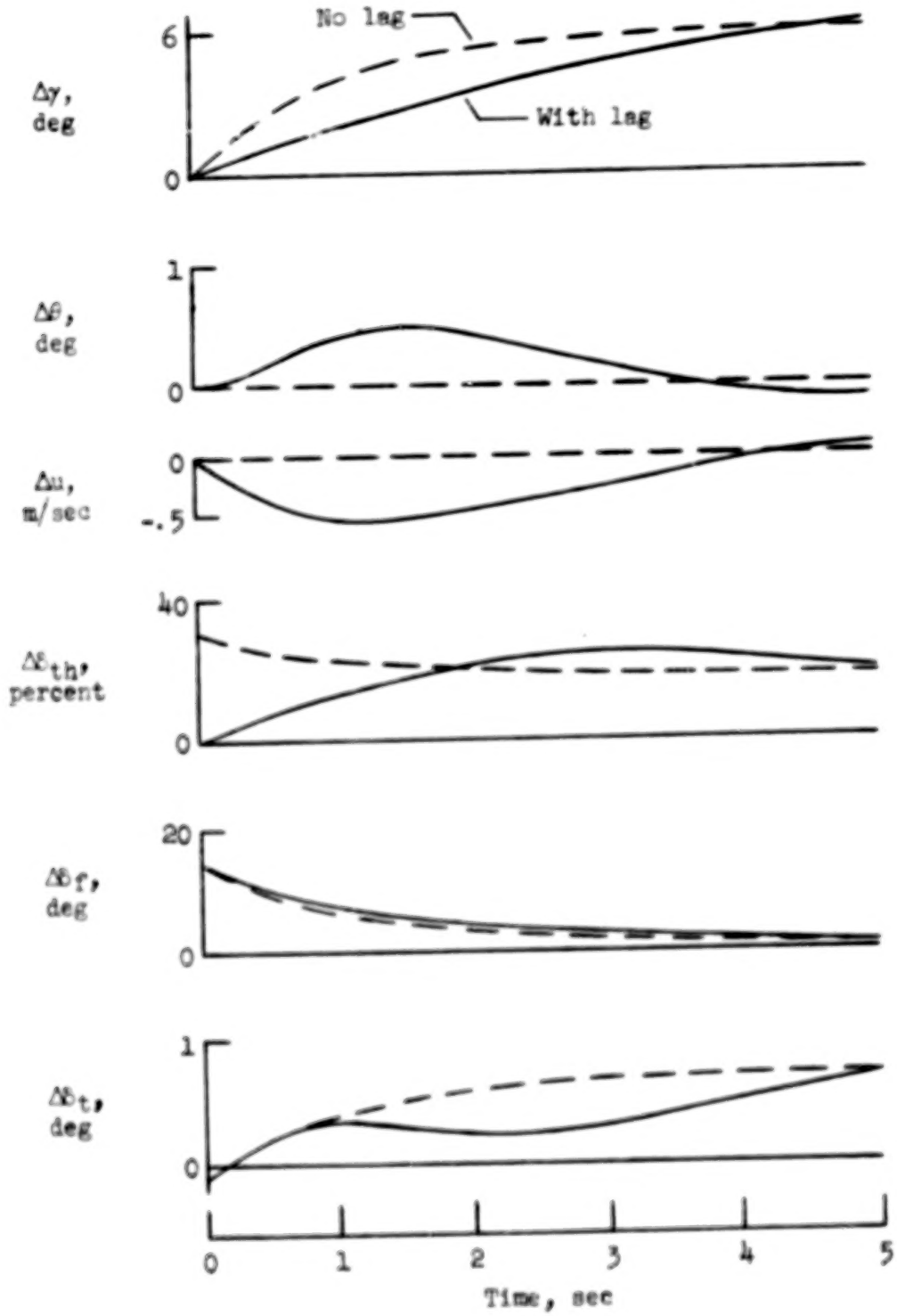
(c) Lag in engine response (τ_{th} = 2.0 sec).

Figure 14.- Concluded.



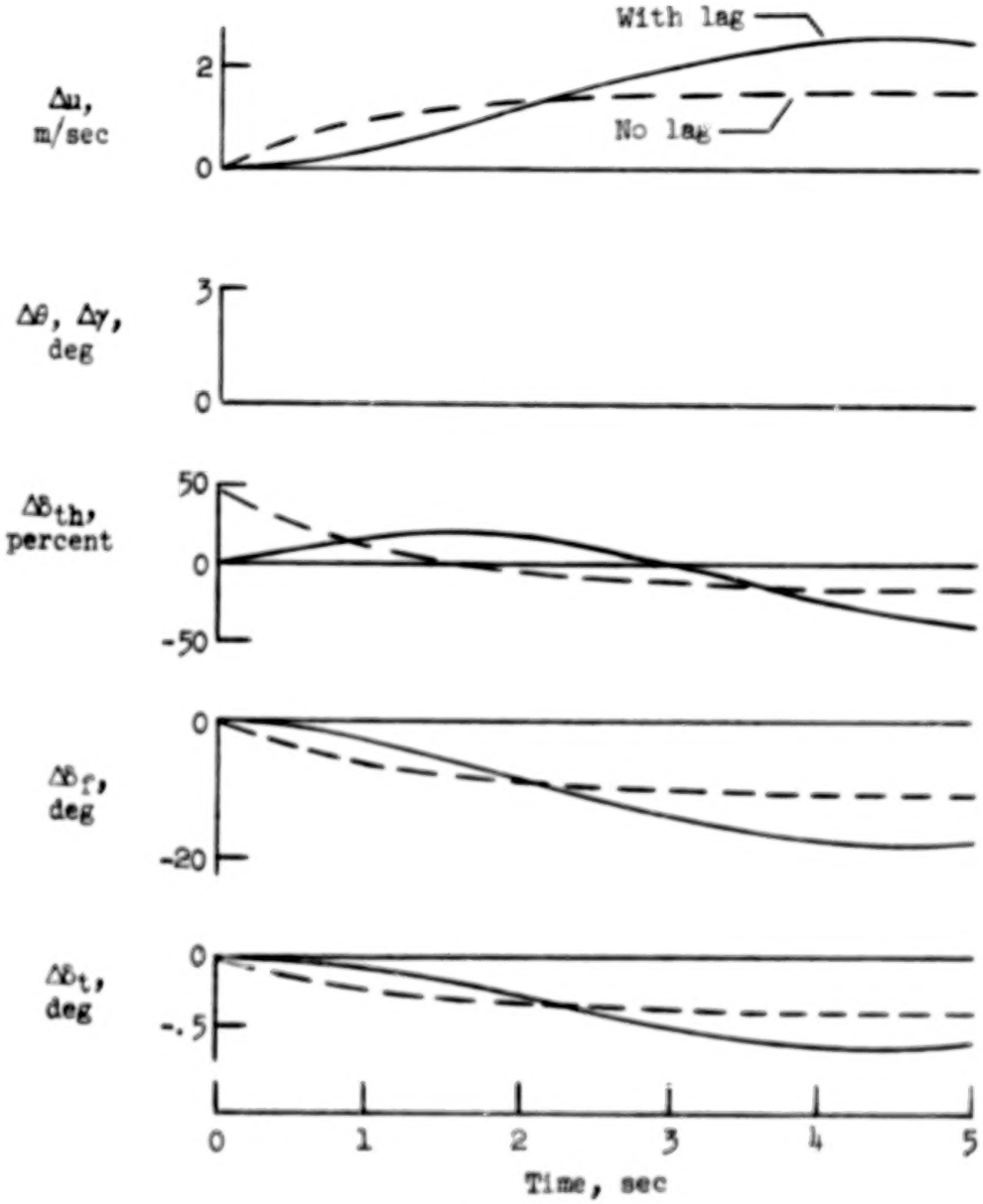
(a) Standard case $\left(C_{z_{\delta_{th}}} \text{ and } C_{m_{\delta_{th}}} = 0\right)$.

Figure 15.- Effect of engine-response lag (τ_{th} = 2.0 sec) for flight-path-angle command of 6°.



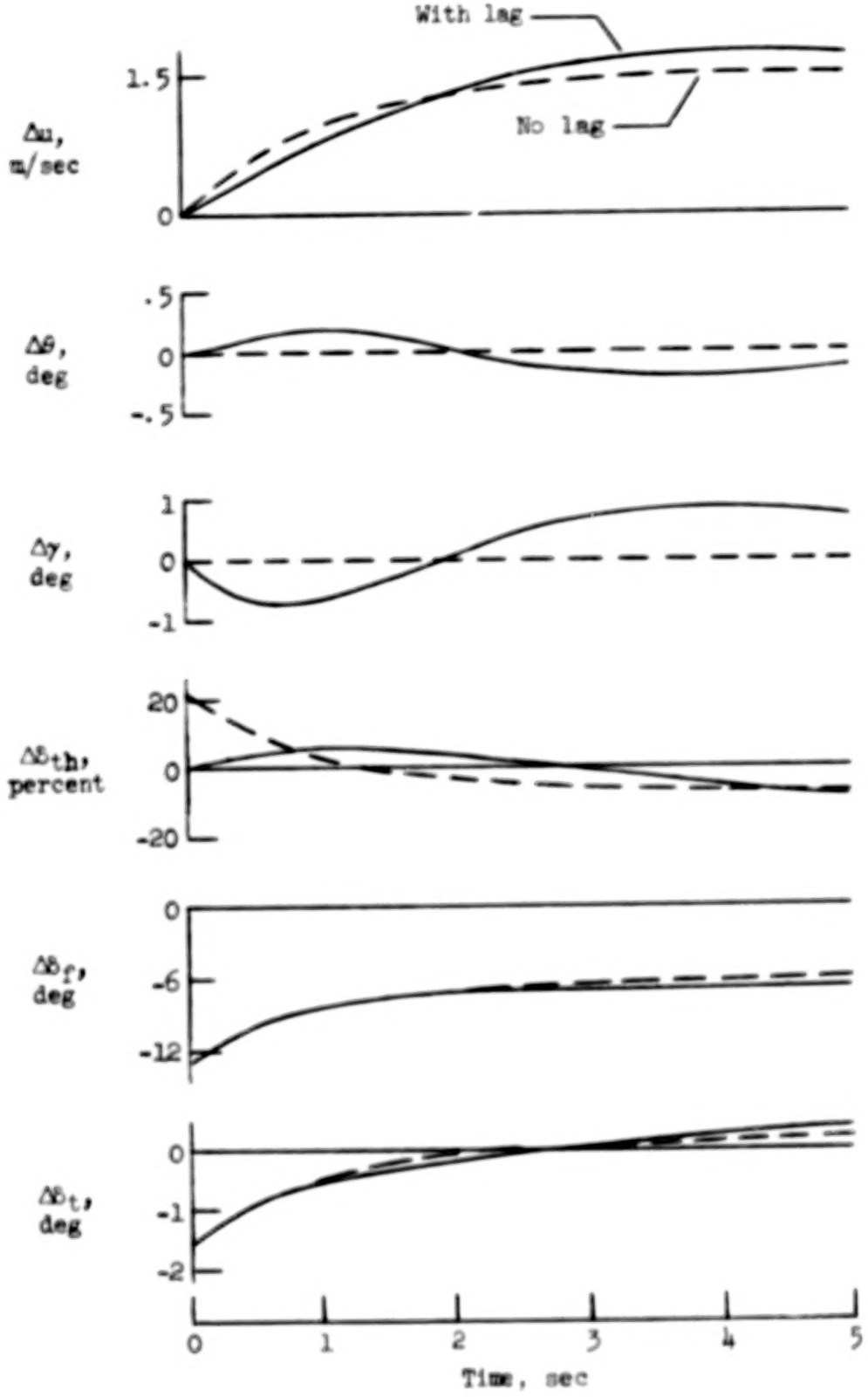
(b) $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$.

Figure 15.- Concluded.



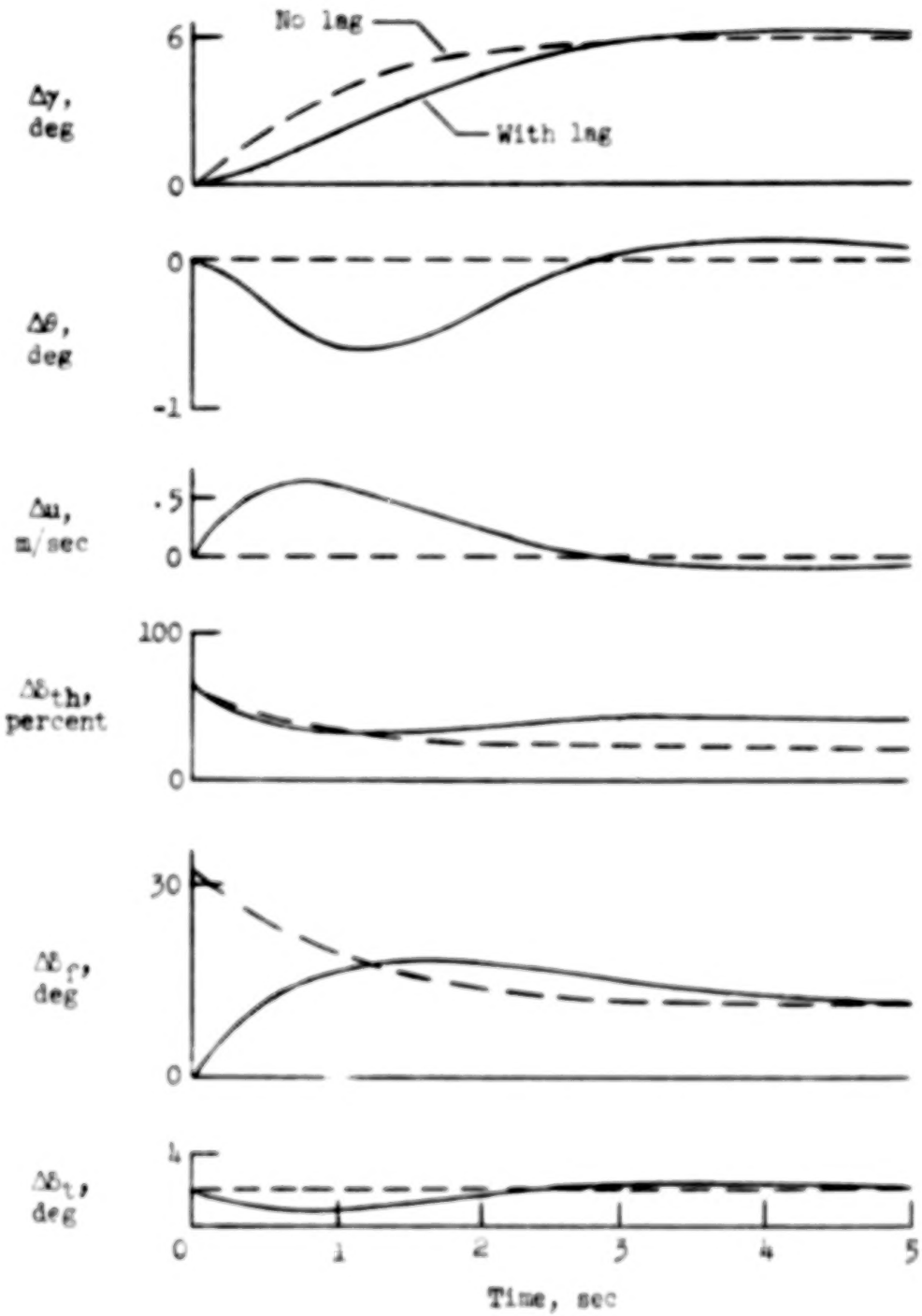
(a) Standard case $\left(C_{Z_{\delta_{th}}} \text{ and } C_{m_{\delta_{th}}} = 0\right)$.

Figure 16.- Effect of engine-response lag (τ_{th} = 2.0 sec) for speed command of 1.5 m/sec.



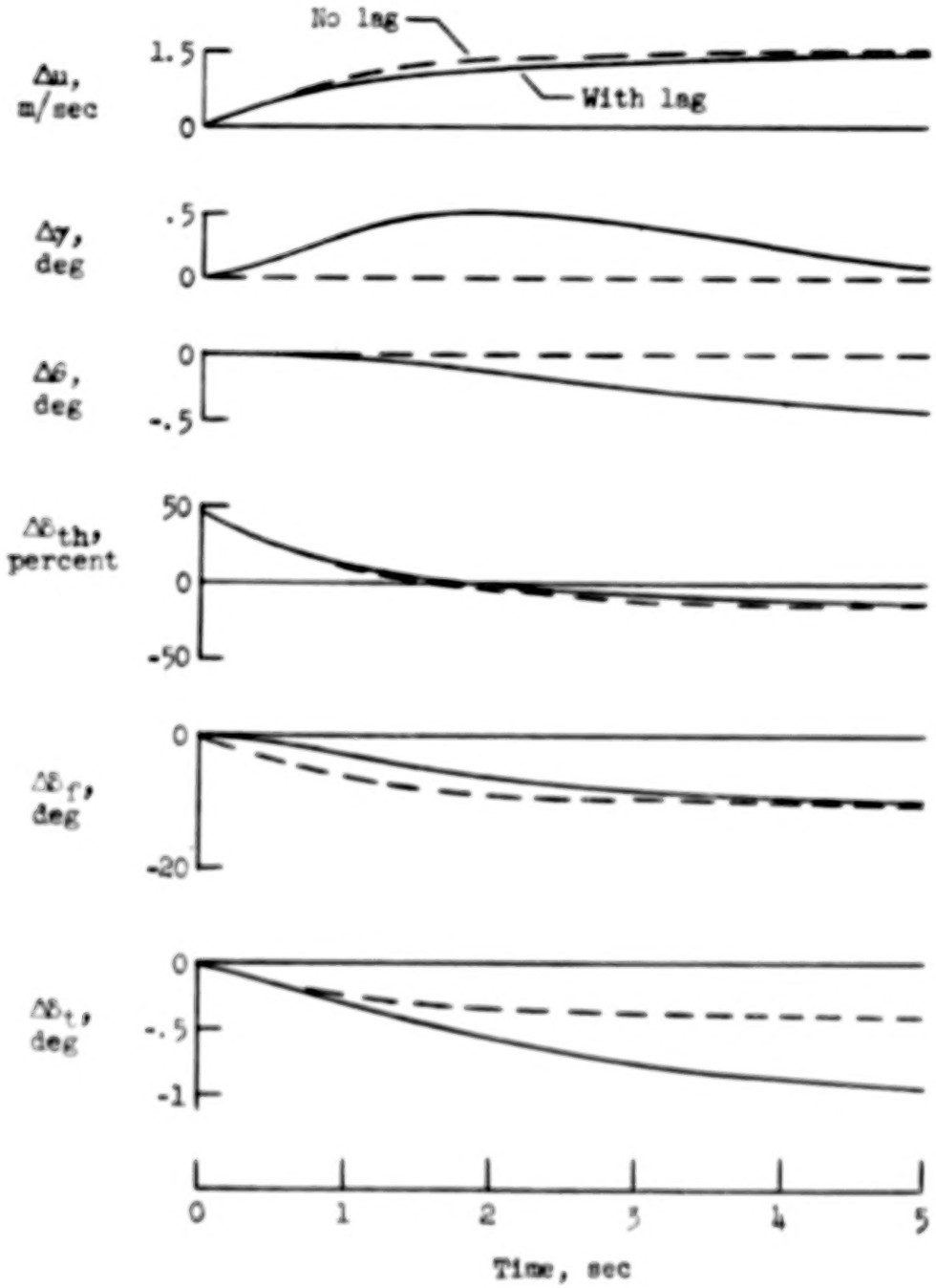
(b) $C_{Z_{\delta_{th}}}$ and $C_{m_{\delta_{th}}} \neq 0$.

Figure 16.- Concluded.



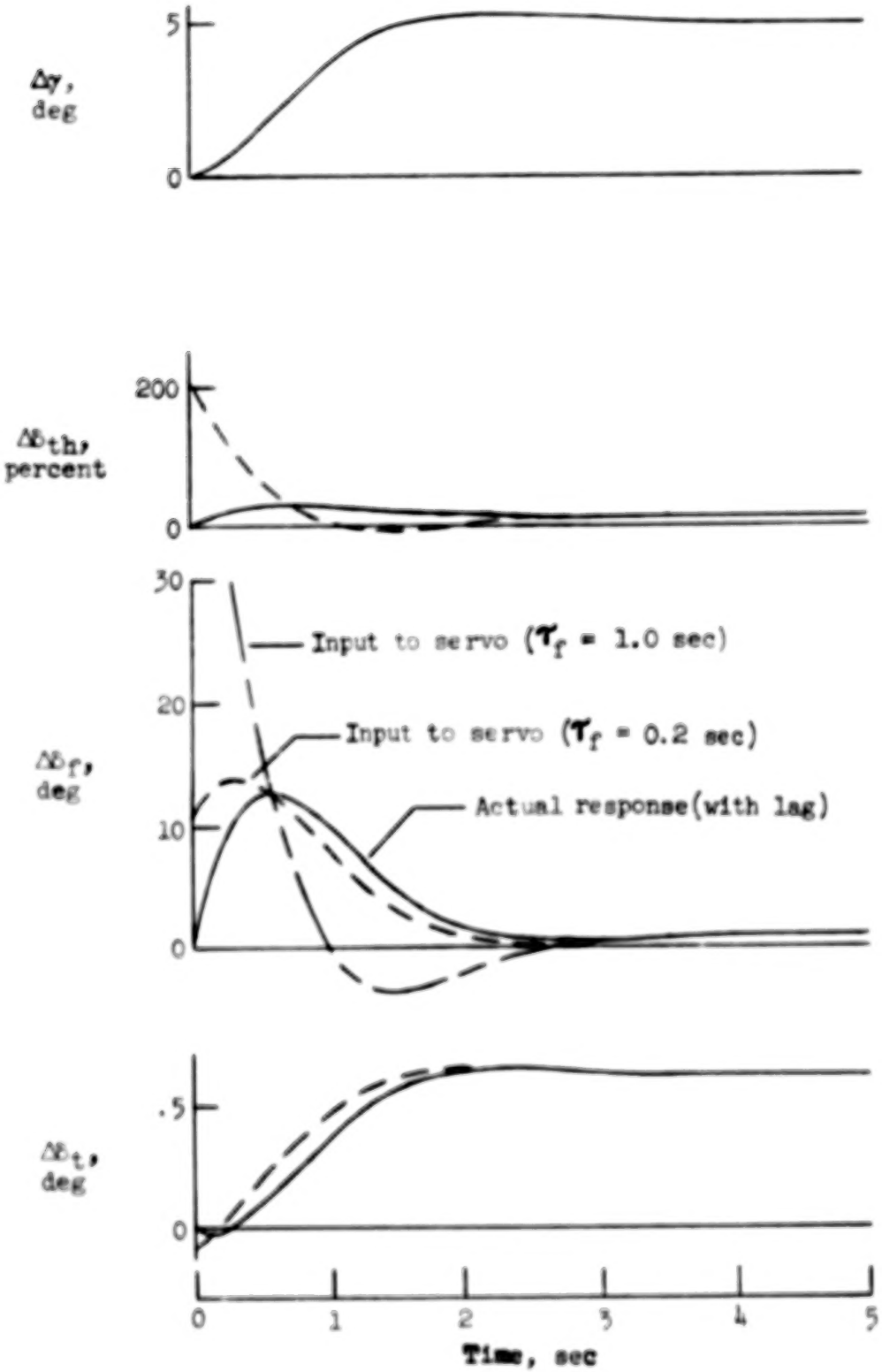
(a) Flight-path-angle command of 6°.

Figure 17.- Effect of flap-servo lag (τ_f = 1.0 sec).



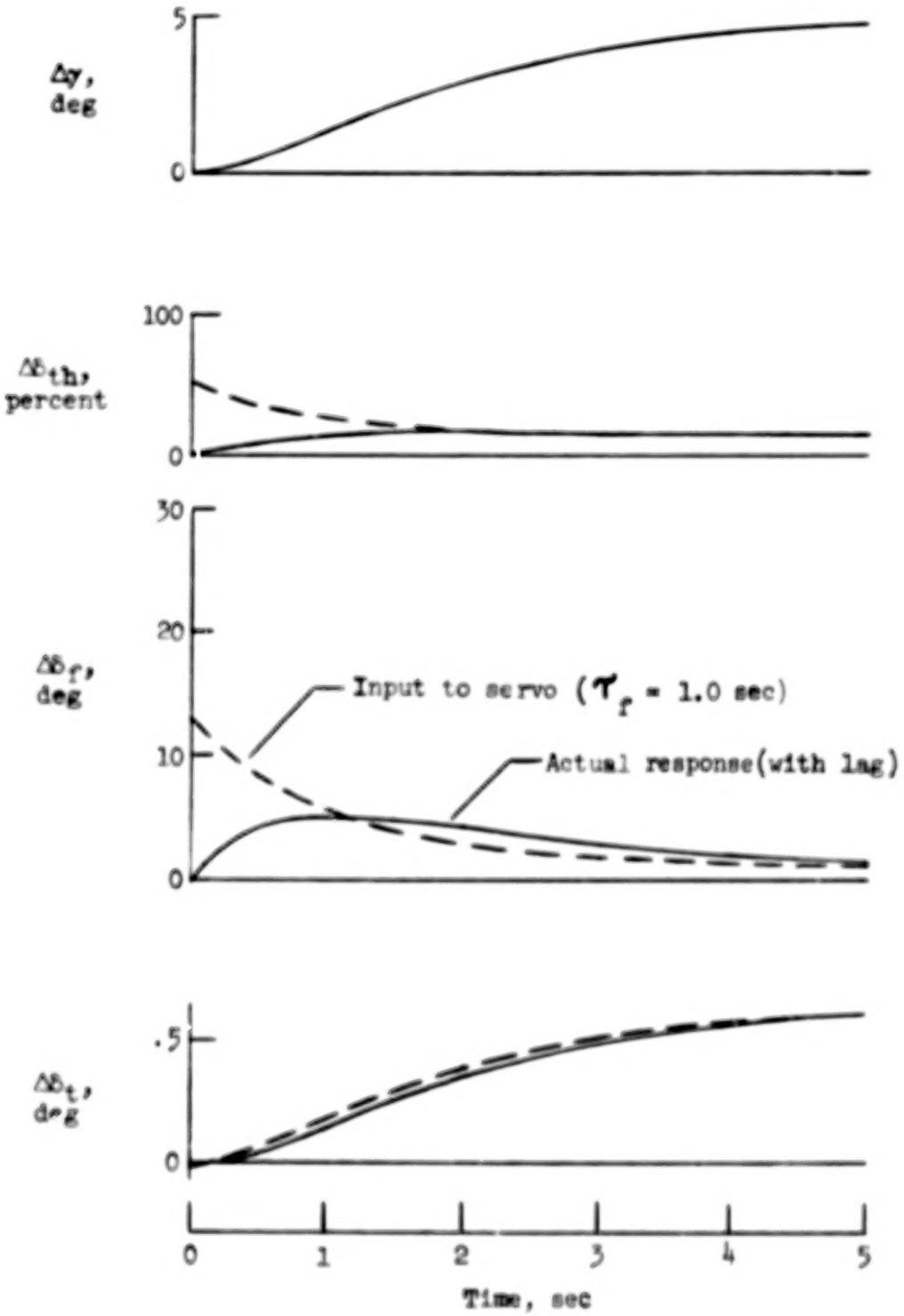
(b) Speed command of 1.5 m/sec.

Figure 17.- Concluded.



(a) ω_3 = 2 rad/sec; ζ_3 = 0.7.

Figure 18.- γ command of 5° with control lag incorporated into design of decoupled control system (τ_t = 0.2 sec; τ_{th} = 2.0 sec).



(b) τ_3 = 1 sec.

Figure 18.- Concluded.

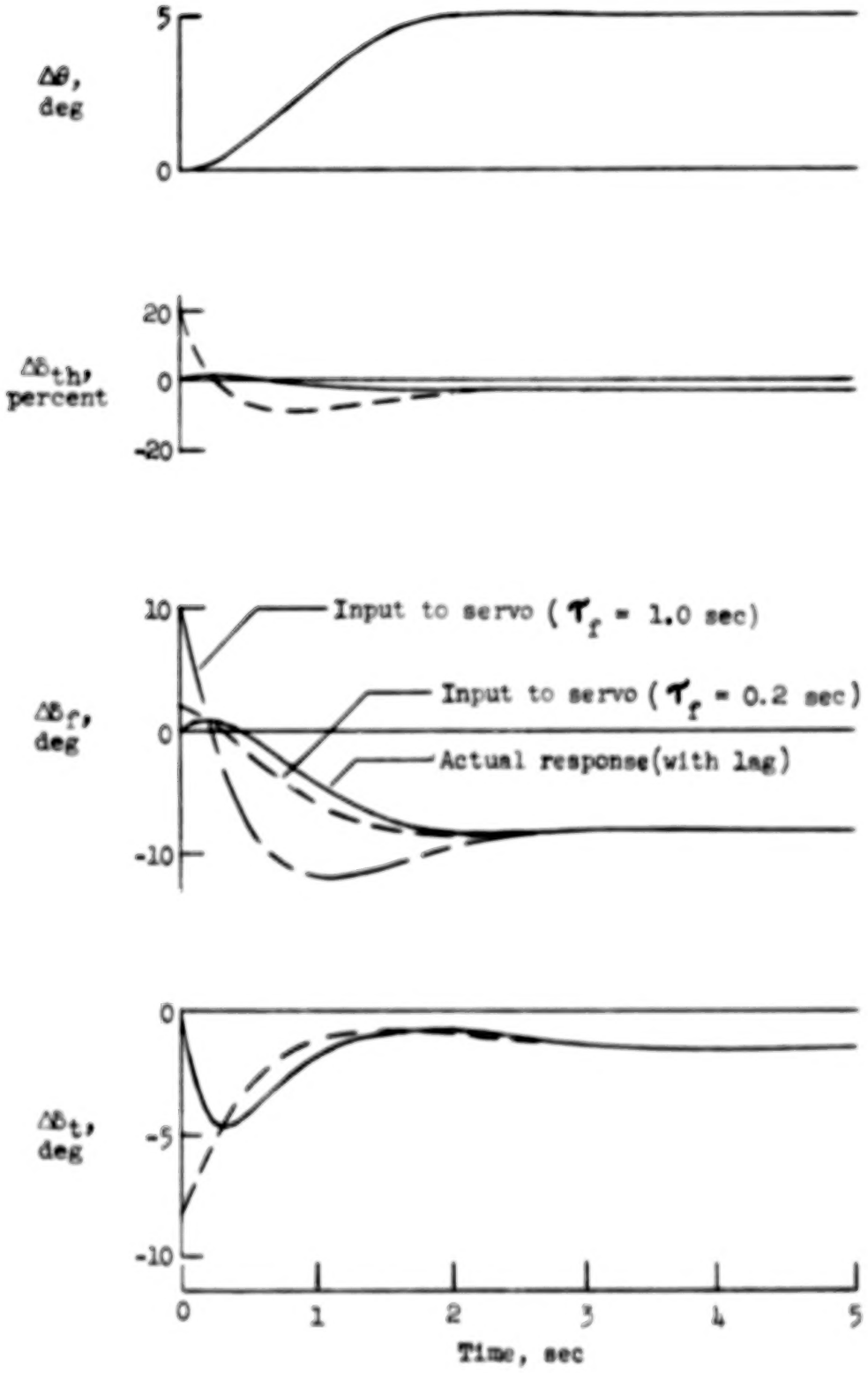
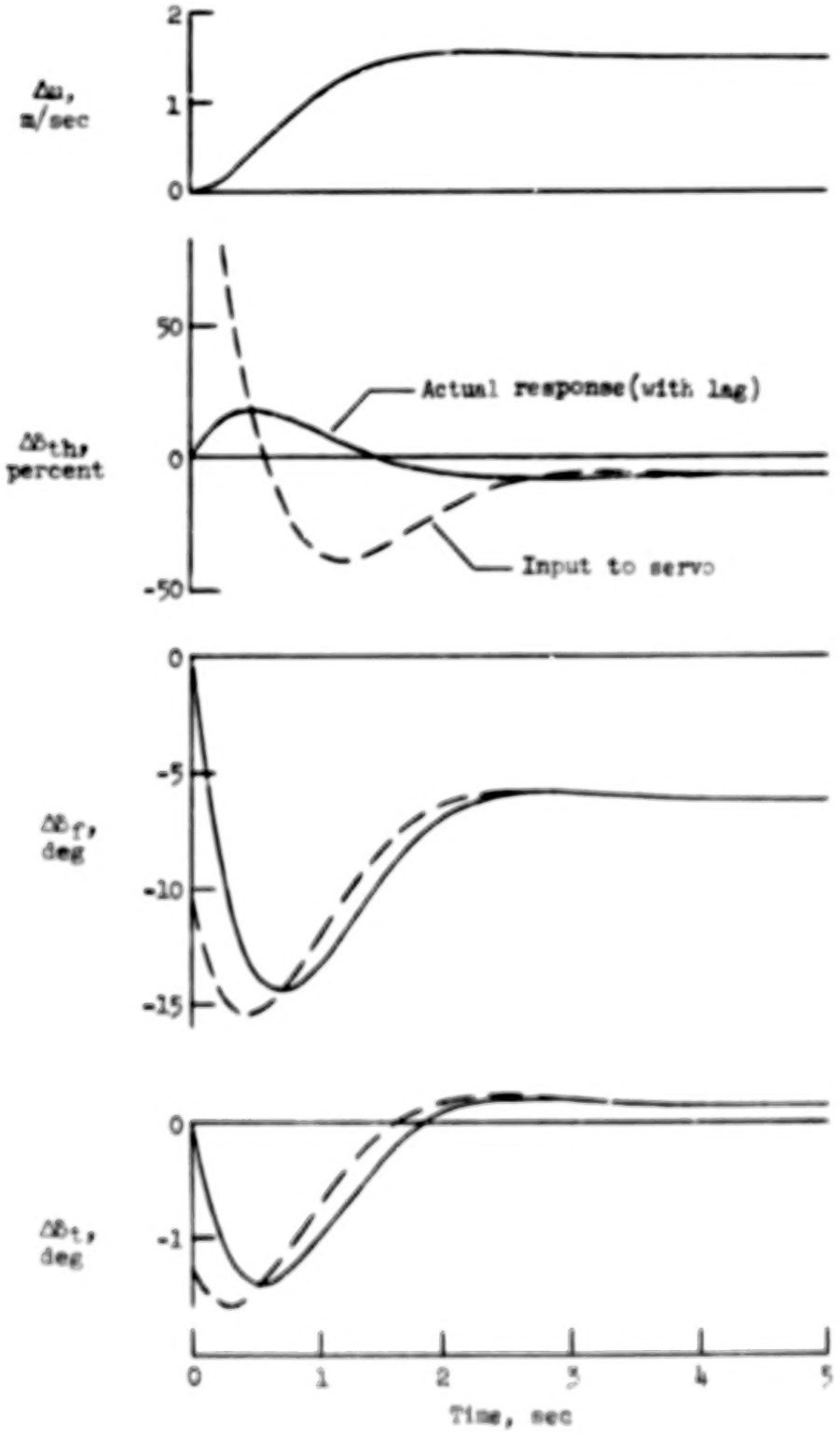
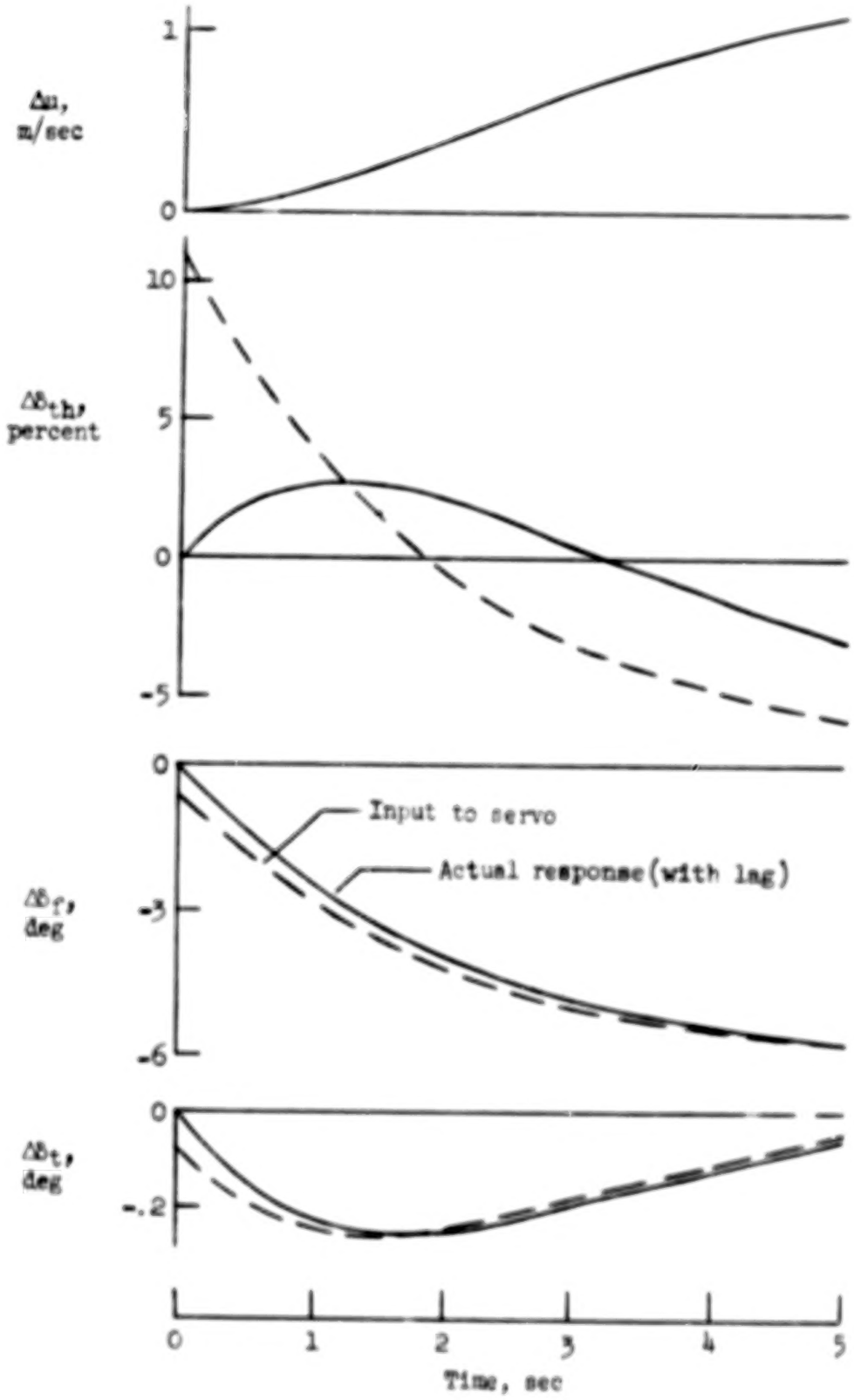


Figure 19.- θ-command maneuver of 5° with control lag incorporated into design of decoupled control system (ω_n = 2 rad/sec; ζ = 0.7; τ_t = 0.2 sec; τ_{th} = 2 sec).



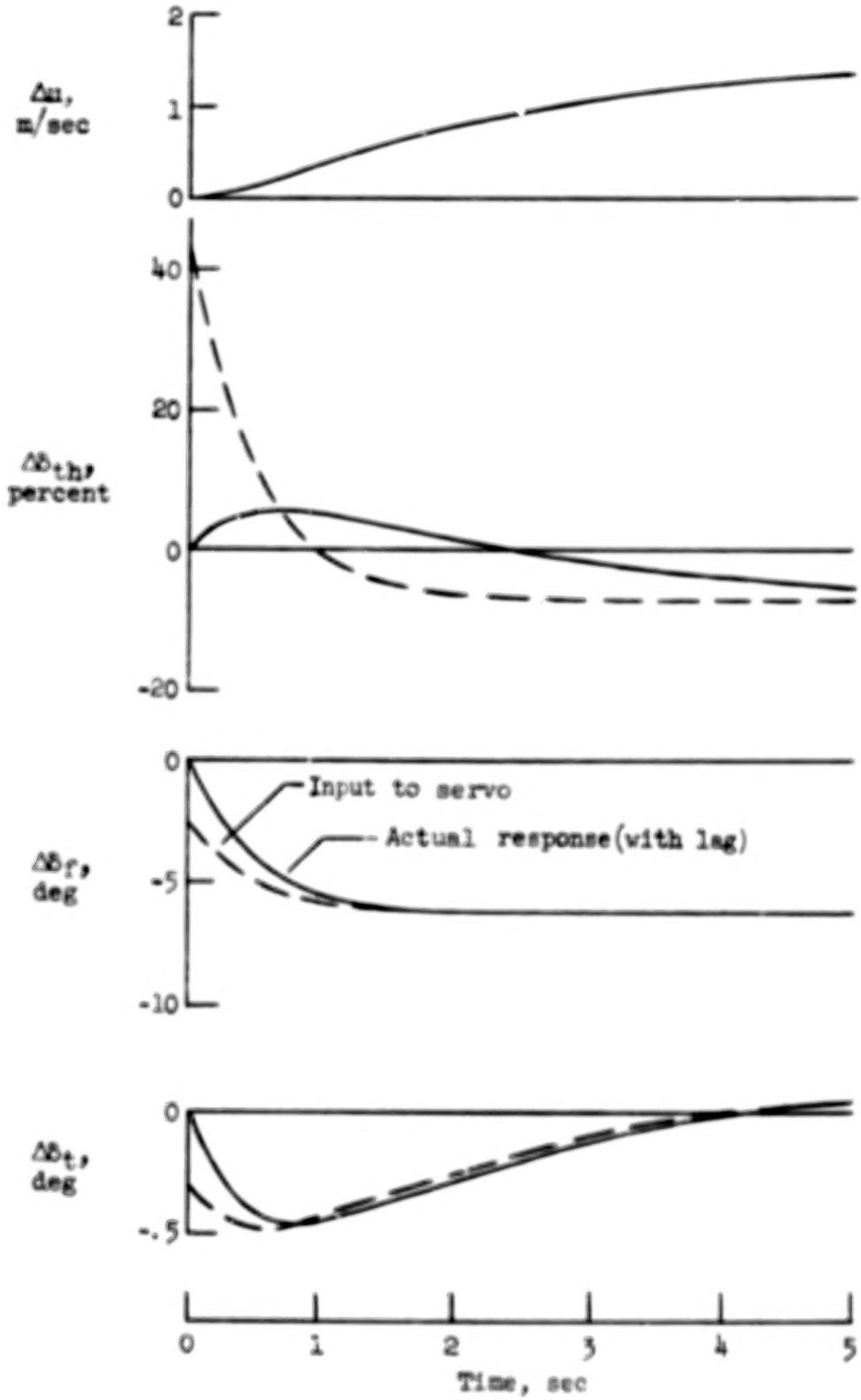
(a) $\omega_1 = 2$ rad/sec; $\zeta_1 = 0.7$.

Figure 20.- Speed-command maneuver of 1.5 m/sec with control lag incorporated into design of decoupled control system ($\tau_f = 0.2$ sec; $\tau_t = 0.2$ sec; $\tau_{th} = 2.0$ sec).



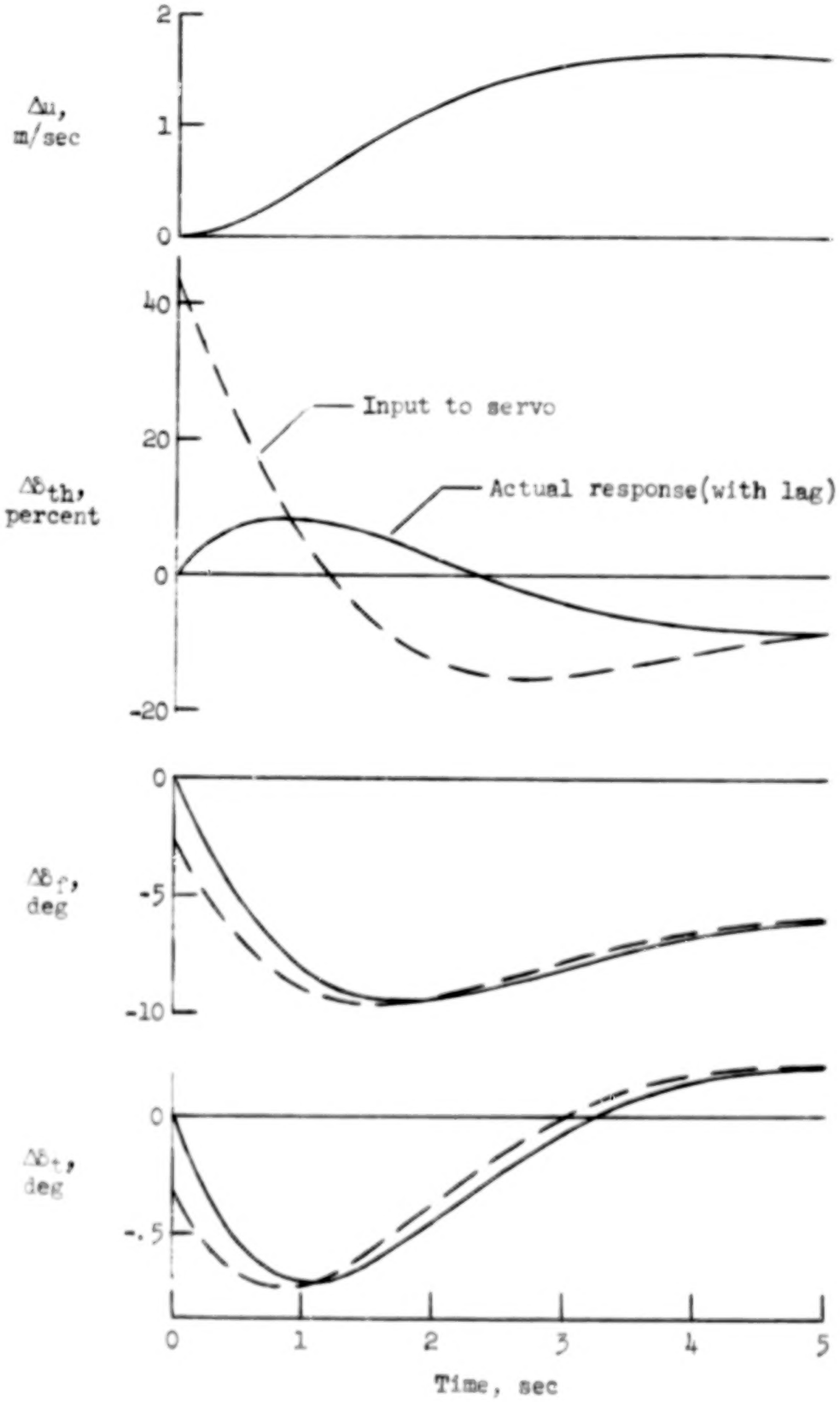
(b) τ_1 = 2.0 sec.

Figure 20.- Continued.



(c) $\tau_1 = 0.5$ sec (or $\omega_1 = 1.0$ rad/sec; $\zeta_1 = 1.25$).

Figure 20.- Continued.



(d) ω_1 = 1.0 rad/sec; ζ_1 = 0.625.

Figure 20.- Concluded.

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15. Supplementary Notes

16. Abstract

A study to determine the effects of various error sources on a decoupled control system was performed in connection with the longitudinal control system on a simulated externally blown jet-flap STOL aircraft. The system employed the throttle, horizontal tail, and flaps to decouple the forward velocity, pitch angle, and flight-path angle. The errors considered were (1) imperfect knowledge of airplane aerodynamic and control characteristics, (2) imperfect measurements of airplane state variables, (3) change in flight conditions, and (4) lag in the airplane controls and in engine response.

Despite the large errors employed in the study, the effects of the various errors on the decoupling process were generally minor. Significant coupling in flight-path angle was caused by control lag during speed-command maneuvers. However, this coupling could be eliminated by including the control lag in the design of the decoupled system. Other error sources affected the commanded response quantity primarily; the control requirements and the other responses were materially unaffected.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
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